SCIENCE COLLEGE (AUTONOMOUS)

HINJILICUT, GANJAM, ODISHA

COURSES OF STUDIES

FOR

MATHEMATICS

First Semester Examination-	**2019-20**
Second Semester Examination-	**2019-20**
Third Semester Examination-	**2020-21**
Fourth Semester Examination-	**2020-21**
Fifth Semester Examination-	**2021-22**
Sixth Semester Examination-	**2021-22**

STATE MODEL SYLLABUS FOR
UNDER GRADUATE
COURSE IN MATHEMATICS
(Bachelor of Science Examination)

UNDER

CHOICE BASED CREDIT SYSTEM

Preamble

Mathematics is an indispensable tool for much of science and engineering. It provides the basic language for understanding the world and lends precision to scientific thought. The mathematics program at Universities of Odisha aims to provide a foundation for pursuing research in Mathematics as well as to provide essential quantitative skills to those interested in related fields. With the maturing of the Indian industry, there is a large demand for people with strong analytical skills and broad-based background in the mathematical sciences.

COURSE STRUCTURE FOR MATHEMATICS HONORS

Semester	Course	Course Name	Credits
I	AECC-I	AECC-I (Environmental Science)	04
	C-I C-I	Calculus Practical	04 02
	C-II C-II	Discrete Mathematics Tutorial	05 01
	GE-I GE-I	GE-I Tutorial	05 01
			22
II	AECC-II	MIL (Odia Communication/ Alternative English)	04
	C-III C-III	Real Analysis Tutorial	05 01
	C-IV C-IV	Differential equations Practical	04 02
	GE-II GE-II	GE-II Tutorial	05 01
			22
III	C-V C-V	Theory of Real functions Tutorial	05 01
	C-VI C-VI	Group Theory-I Tutorial	05 01
	C-VII	Partial differential equations and system of ODEs	04

	C-VII	Practical	02
	GE-III GE-III	GE-III Tutorial	05 01
	SECC-I	SECC-I Communicative English	04
			28
IV	C-VIII C-VIII	Numerical Methods and Scientific Computing Practical	04 02
	C-IX C-IX	Topology of Metric spaces Tutorial	05 01
	C-X C-X	Ring Theory Tutorial	05 01
	GE-IV GE-IV	GE-IV (Theory) Tutorial	05 01
	SECC-II	SECC-II Quantitative & Logical Thinking	04
			28
Semester	Course	Course Name	Credits
V	C-XI C-XI	Multivariable Calculus Tutorial	05 01
	C-XII C-XII	Linear Algebra Tutorial	05 01
	DSE-I DSE-I	Linear Programming Tutorial	05 01
	DSE-II DSE-II	Probability and Statistics Tutorial	05 01

			24
VI	C-XIII C-XIII	Complex analysis Tutorial	05 01
	C-XIV C-XIV	Group Theory-II Tutorial	05 01
	DSE-III DSE-III	Differential Geometry Tutorial	05 01
	DSE-IV	Number Theory/Project	06
			24
		TOTAL	**148**

B.A./B.SC.(HONOURS)-MATHEMATICS

HONOURS PAPERS:

Core course – 14 papers

Discipline Specific Elective – 4 papers (out of the 5 papers suggested)

Generic Elective for non Mathematics students – 4 papers. Incase University offers 2 subjects as GE, then papers 1 and 2 will be the GE paper.

Marks per paper –

For practical paper: Midterm : 15 marks, End term : 60 marks, Practical- 25 marks

For non practical paper: Mid term : 20 marks, End term : 80 marks

Total – 100 marks Credit per paper – 6

Teaching hours per paper –

Practical paper-40 hour theory classes + 20 hours Practical classes

Non Practical paper-50 hour theory classes + 10 hours tutorial

CORE PAPER-1

CALCULUS

Objective: The main emphasis of this course is to equip the student with necessary analytic and technical skills to handle problems of mathematical nature as well as practical problems. More precisely, main target of this course is to explore the different tools for higher order derivatives, to plot the various curves and to solve the problems associated with differentiation and integration of vector functions.

Excepted Outcomes: After completing the course, students are expected to be able to use Leibnitz's rule to evaluate derivatives of higher order, able to study the geometry of various types of functions, evaluate the area, volume using the techniques of integrations, able to identify the difference between scalar and vector, acquired knowledge on some the basic properties of vector functions.

UNIT-I

Hyperbolic functions, higher order derivatives, Leibnitz rule and its applications to problems of the type $e^{as+b}\sin x$, $e^{as+b}\cos x$, $(ax+b)^n\sin x$, $(ax+b)^n\cos x$, concavity and inflection points, asymptotes, curve tracing in Cartesian coordinates, tracing in polar coordinates of standard curves, L' Hospitals rule, Application in business ,economics and life sciences.

UNIT-II

Riemann integration as a limit of sum, integration by parts, Reduction formulae, derivations and illustrations of reduction formulae of the type $\int \sin^n x dx$, $\int \cos^n x dx$, $\int \tan^n x dx$, $\int \sec^n x dx$, $\int (\log x)^n dx$, $\int \sin^n x \cos^n x dx$,

definite integral, integration by substitution.

UNIT-III

Volumes by slicing, disks and washers methods, volumes by cylindrical shells, parametric equations, parameterizing a curve, arc length, arc length of parametric curves, area of surface of revolution, techniques of sketching conics, reflection properties of conics, rotation of axes and second degree equations, classification into conics using the discriminant, polar equations of conics.

UNIT-IV

Triple product, introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions, tangent and normal components of acceleration.

LIST OF PRACTICALS

(To be performed using Computer with aid of MATLAB or such software)

1. Plottingthe graphsofthe functionse^{as+b}, $\log(ax+b)$, $1/ax+b$, $\sin(ax+b)$, $\cos(ax+b)$ and $|ax+b|$to illustrate the effect of a and b on the graph.

2. Plotting the graphs of the polynomial of degree 4 and5.
3. Sketching parametric curves (E.g. Trochoid, cycloid, hypocycloid).
4. Obtaining surface of revolution of curves.
5. Tracing of conics in Cartesian coordinates/polar coordinates.
6. Sketching ellipsoid, hyperboloid of one and two sheets (using Cartesian co-ordinates).

BOOKS RECOMMENDED:

1. H.Anton, I.Bivensand S.Davis, *Calculus*,10thEd., JohnWileyand Sons(Asia) P.Ltd., Singapore, 2002.
2. Shanti Narayan, P. K. Mittal, Differential Calculus, S. Chand, 2014.
3. Shanti Narayan, P. K. Mittal, Integral Calculus, S. Chand, 2014.

BOOKS FOR REFERNCE:

1. James Stewart, Single Variable Calculus, Early Transcendentals, Cengage Learning, 2016.
2. G.B. Thomas and R.L. Finney, *Calculus*, 9th Ed., Pearson Education, Delhi,2005.

CORE PAPER-II

DISCRETE MATHEMATICS

Objective: This is a preliminary course for the basic courses in mathematics and all its applications. The objective is to acquaint students with basic counting principles, set theory and logic, matrix theory and graph theory.

Expected Outcomes: The acquired knowledge will help students in simple mathematical modeling. They can study advance courses in mathematical modeling, computer science, statistics, physics, chemistry etc.

UNIT-I

Sets, relations, Equivalence relations, partial ordering, well ordering, axiom of choice, Zorn's lemma, Functions, cardinals and ordinals, countable and uncountable sets, statements, compound statements, proofs in Mathematics, Truth tables, Algebra of propositions, logical arguments, Well-ordering property of positive integers, Division algorithm, Divisibility and Euclidean algorithm, Congruence relation between integers, modular arithmetic, Chinese remainder theorem, Fermat's little theorem.

UNIT-II

Principles of Mathematical Induction, pigeonhole principle, principle of inclusion and exclusion Fundamental Theorem of Arithmetic, permutation combination circular permutations binomial and multinomial theorem, Recurrence relations, generating functions, generating function from recurrence relations.

UNIT-III

Matrices, algebra of matrices, determinants, fundamental properties, minors and cofactors, product of determinant, adjoint and inverse of a matrix, Rank and nullity of a matrix, Systems of linear equations, row reduction and echelon forms, solution sets of linear systems, applications of linear systems, Eigen values, Eigen vectors of amatrix.

UNIT-IV

Graph terminology, types of graphs, subgraphs, isomorphic graphs, Adjacency and incidence matrices, Paths, Cycles and connectivity, Eulerian and Hamiltonian paths, Planar graphs.

BOOKS RECOMMENDED:

1. Edgar G. Goodaire and Michael M. Parmenter, Discrete Mathematics with Graph Theory, 3rd Ed., Pearson Education (Singapore) P. Ltd., Indian Reprint, 2005.

2. Kenneth Rosen Discrete mathematics and its applications Mc Graw Hill Education 7th edition.
3. V Krishna Murthy, V. P. Mainra, J. L. Arora, An Introduction to Linear Algebra, Affiliated East-West Press Pvt. Ltd.

BOOKS FOR REFERENCE:

1. J. L. Mott, A. Kendel and T.P. Baker: Discrete mathematics for Computer Scientists and Mathematicians, Prentice Hall of India Pvt Ltd, 2008.

CORE PAPER-III

REAL ANALYSIS

Objective: The objective of the course isto have the knowledge on basic properties of the field of real numbers, studying Bolzano-Weierstrass Theorem , sequences and convergence of sequences, series of real numbers and its convergence etc. This is one of the core courses essential to start doing mathematics.

Expected Outcome: On successful completion of this course, students will be able to handle fundamental properties of the real numbers that lead to the formal development of real analysis and understand limits and their use in sequences, series, differentiation and integration. Students will appreciate how abstract ideas and rigorous methods in mathematical analysis can be applied to important practical problems.

UNIT-I

Review of Algebraic and Order Properties of *R,* s-neighborhood of a point in *R,* Bounded above sets, Bounded below sets, Bounded Sets, Unbounded sets, Suprema and Infima, The Completeness Property of *R,* The Archimedean Property, Density of Rational (and Irrational) numbers in *R.,* Intervals, Interior point, , Open Sets, Closed sets, , Limit points of a set , Illustrations of Bolzano-Weierstrass theorem for sets, closure, interior and boundary of a set.

UNIT-II

Sequences and Subsequences, Bounded sequence, Convergent sequence, Limit of a sequence.

Limit Theorems, Monotone Sequences,. Divergence Criteria, Bolzano Weierstrass Theorem for Sequences, Cauchy sequence, Cauchy's Convergence Criterion. Infinite series, convergence and divergence of infinite series, Cauchy Criterion, Tests for convergence: Comparison test, Limit Comparison test, Ratio Test, Cauchy's nth root test, Integral test, Alternating series, Leibniz test, Absolute and Conditional convergence.

UNIT-III

Limitsof functions (epsilon-deltaapproach),sequential criterionforlimits, divergence criteria. Limit theorems, onesidedlimits, Infinitelimitsandlimits at infinity, Continuous functions, sequential criterion forcontinuity &discontinuity. Algebra ofcontinuousfunctions, Continuousfunctions onaninterval, Boundedness Theorem, Maximum Minimum Theorem, Bolzano's Intermediatevaluetheorem, location of rootstheorem, preservation of intervalstheorem. Uniform continuity, non-uniform continuity criteria, uniform continuity theorem, Monotone and Inverse Functions.

UNIT-IV

Differentiabilityofafunction ata point&inaninterval, Caratheodory'stheorem, chain Rule,algebra of differentiable functions, Mean value theorem, interior extremum theorem. Rolle'stheorem, intermediate value property ofderivatives,Darboux'stheorem. Applications of mean value theorem toinequalities.

BOOKS RECOMMENDED:

1. R.G. Bartle and D. R. Sherbert, Introduction to Real Analysis(3rd Edition), John Wiley and Sons (Asia) Pvt. Ltd., Singapore,2002.
2. G. Das and S. Pattanayak, Fundamentals of Mathematical Analysis, TMH Publishing Co.

BOOKS FOR REFERENCE:

1. S.C. Mallik and S. Arora-Mathematical Analysis, New Age InternationalPublications.
2. A.Kumar, S. Kumaresan, *A basic course in Real Analysis*, CRC Press, 2014.
3. BrianS.*Thomson*,Andrew.M.*Bruckner*,andJudithB.*Bruckner*,*ElementaryReal Analysis,* Prentice Hall,2001.
4. Gerald G. Bilodeau, Paul R. Thie, G.E. Keough, *An Introductionto Analysis,*Jones & Bartlett, Second Edition, 2010.

CORE PAPER-IV

DIFFERENTIAL EQUATIONS

Objective: Differential Equations introduced by Leibnitz in 1676 models almost all Physical, Biological, Chemical systems in nature. The objective of this course is to familiarize the students with various methods of solving differential equations and to have a qualitative applications through models. The students have to solve problems to understand the methods.

Expected Outcomes: A student completing the course is able to solve differential equations and is able to model problems in nature using Ordinary Differential Equations. This is also prerequisite for studying the course in Partial Differential Equations and models dealing with Partial Differential Equations.

UNIT-I

Differential equations and mathematical models, General, Particular, explicit, implicit and singular solutions of a differential equation. Exact differential equations and integrating factors, separable equations and equations reducible to this form, linear equations and Bernoulli's equation, special integrating factors and transformations.

UNIT-II

Introduction to compartmental models, Exponential decay radioactivity (case study of detecting art forgeries), lake pollution model (with case study of Lake Burley Griffin), drug assimilation into the blood (case study of dull, dizzy and dead), exponential growth of population, Density dependent growth, Limited growth with harvesting.

UNIT-III

General solution of homogeneous equation of second order, principle of superposition, Wronskian, its properties and applications, method of undetermined coefficients, Method of variation of parameters, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation.

UNIT-IV

Equilibrium points, Interpretation of the phase plane, predatory-pray model and its analysis, epidemic model of influenza and its analysis, battle model and its analysis.

Practical / Lab work to be performed on a computer:

Modeling of the following problems using *Matlab / Mathematica / Maple* etc.

1. Plotting of second & third order solution family of differentialequations.
2. Growth & Decay model (exponential caseonly).
3. (a) Lake pollution model (with constant/seasonal flow and pollution concentration)/
   (b) Case of single cold pill and a course of cold pills.
   (c) Limited growth of population (with and without harvesting).
4. (a) Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
   (b) Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
   (c) Battle model (basic battle model, jungle warfare, long range weapons).
5. Plotting of recursivesequences.

BOOKS RECOMMENDED:

1. J. Sinha Roy and S Padhy: A course of Ordinary and Partial differential equation Kalyani Publishers,New Delhi.
2. Belinda Barnes and Glenn R. Fulford, *Mathematical Modeling with Case Studies,A DifferentialEquationApproachusingMapleandMatlab,* 2ndEd.,TaylorandFrancisgroup, London and New York,2009.

BOOKS FOR REFERENCE:

1. Simmons G F, Differential equation, Tata Mc GrawHill, 1991.
2. Martin Braun, Differential Equations and their Applications, Springer International, Student

Ed.

3. S. L. Ross, Differential Equations, 3rd Edition, John Wiley and Sons, India.

4. C.Y. Lin, Theory and Examples of Ordinary Differential Equations, World Scientific, 2011.

CORE PAPER-V
THEORY OF REAL FUNCTIONS

Objective: The objective of the course is to have knowledge on limit theorems on functions, limits of functions, continuity of functions and its properties, uniform continuity, differentiability of functions, algebra of functions and Taylor's theorem and, its applications. The student how to deal with real functions and understands uniform continuity, mean value theorems also.

Expected Outcome: On the completion of the course, students will have working knowledge on the concepts and theorems of the elementary calculus of functions of one real variable. They will work out problems involving derivatives of function and their applications. They can use derivatives to analyze and sketch the graph of a function of one variable, can also obtain absolute value and relative extrema of functions. This knowledge is basic and students can take all other analysis courses after learning this course.

UNIT-I

L' Hospital's Rules, other Intermediate forms, Cauchy's meanvalue theorem, Taylor's theorem with Lagrange's form of remainder, Taylor's theorem with Cauchy's form of remainder, application of Taylor's theorem to convex functions, Relative extrema,Taylor's series andMaclaurin's series, expansions of exponential andtrigonometric functions.

UNIT-II

Riemann integration; inequalities of upper and lower sums; Riemann conditions of integrability. Riemann sum and definition of Riemann integral through Riemann sums; equivalence of two definitions; Riemann integrability of monotone and continuous functions; Properties of the Riemann integral; definition and integrability of piecewise continuous and monotone functions.

Intermediate Value theorem for Integrals; Fundamental theorems of Calculus.

UNIT-III

Improper integrals: Convergence of Beta and Gamma functions. Pointwise and uniform convergence of sequence of functions, uniform convergence, Theorems on continuity, derivability and integrability of the limit function of a sequence of functions.

UNIT-IV

Series of functions; Theorems on the continuity and derivability of the sum function of a series of functions; Cauchy criterion for uniform convergence and Weierstrass M-Test Limit superior and Limit inferior, Power series, radius of convergence, Cauchy Hadamard Theorem, Differentiation and integration of power series; Abel's Theorem; Weierstrass Approximation Theorem.

BOOKS RECOMMENDED:

1. R.G. Bartle & D. R. Sherbert, Introduction to Real Analysis, John Wiley&Sons.
2. G. Das and S. Pattanayak, *Fundamentals of mathematics analysis*, TMH Publishing Co.
3. S. C. Mallik and S. Arora, *Mathematical analysis*, New Age International Ltd., New Delhi.

BOOK FOR REFERENCES:

1. A. Kumar, S. Kumaresan, *A basic course in Real Analysis*, CRC Press, 2014
2. K. A. Ross, *Elementary analysis: the theory of calculus*, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004A.Mattuck, Introduction toAnalysis, Prentice Hall
3. Charles G. Denlinger, *Elements of real analysis*, Jones and Bartlett (Student Edition), 2011.

CORE PAPER-VI

GROUP THEORY-I

Objective: Group theory is one of the building blocks of modern algebra. Objective of this course is to introduce students to basic concepts of group theory and examples of groups and their properties. This course will lead to future basic courses in advanced mathematics, such as Group theory-II and ring theory.

Expected Outcomes: A student learning this course gets idea on concept and examples of groups and their properties . He understands cyclic groups, permutation groups, normal subgroups and related results. After this course he can opt for courses in ring theory, field theory, commutative algebras, linear classical groups etc. and can be apply this knowledge to problems in physics, computer science, economics and engineering.

UNIT-I

Symmetries of a square, Dihedral groups, definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), elementary properties of groups, Subgroups and examples of subgroups, centralizer, normalizer, center of a group,

UNIT-II

Product of two subgroups, Properties of cyclic groups, classification of subgroups of cyclic groups,Cycle notation for permutations, properties of permutations, even and odd permutations, alternating group,

UNIT-III

Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem, external direct product of a finite number of groups, normal subgroups, factor groups.

UNIT-IV

Cauchy's theorem for finite abelian groups, group homomorphisms, properties of homomorphisms, Cayley's theorem, properties of isomorphisms, first, second and third isomorphism theorems.

BOOKS RECOMMENDED:

1. Joseph A. Gallian, *Contemporary Abstract Algebra* (4th Edition), Narosa Publishing House, New Delhi
2. John B. Fraleigh, *A First Course in Abstract Algebra,* 7th Ed., Pearson, 2002.

BOOK FOR REFERENCES:

1. M. *Artin, Abstract Algebra,* 2nd Ed., Pearson, 2011.
2. Joseph 1. Rotman, *An Introduction to the Theory of Groups,* 4th Ed., Springer Verlag, 1995.
3. I. N. Herstein, *Topics in Algebra,* Wiley Eastern Limited, India, 1975.

CORE PAPER-VII

PARTIAL DIFFERENTIAL EQUATIONS AND SYSTEM OF ODEs

Objective: The objective of this course is to understand basic methods for solving Partial Differential Equations of first order and second order. In the process, students will be exposed to Charpit's Method, Jacobi Method and solve wave equation, heat equation, Laplace Equation etc. They will also learn classification of Partial Differential Equations and system of ordinary differential equations.

Expected Outcomes: After completing this course, a student will be able to take more courses on wave equation, heat equation, diffusion equation, gas dynamics, non linear evolution equations etc. All these courses are important in engineering and industrial applications for solving boundary value problem.

UNIT-I

Partial Differential Equations - Basic concepts and Definitions, Mathematical Problems. First-Order Equations: Classification, Construction and Geometrical Interpretation. Method of Characteristics for obtaining General Solution of Quasi Linear Equations. Canonical Forms of First-order Linear Equations. Method of Separation of Variables for solving first order partial differential equations.

UNIT-II

Derivation of Heat equation, Wave equation and Laplace equation. Classification of second order linear equations as hyperbolic, parabolic or elliptic. Reduction of second order Linear Equations to canonical forms.

UNIT-III

The Cauchy problem, Cauchy problem of an infinite string. Initial Boundary Value Problems, Semi-Infinite String with a fixed end, Semi-Infinite String with a Free end. Equations with non-homogeneous boundary conditions, Non- Homogeneous Wave Equation. Method of separation of variables, Solving the Vibrating String Problem, Solving the Heat Conduction problem

UNIT-IV

Systems of linear differential equations, types of linear systems, differential operators, an operator method for linear systems with constant coefficients, Basic Theory of linear systems in normal form, homogeneous linear systems with constant coefficients: Two Equations in two unknown functions, The method of successive approximations.

LIST OF PRACTICALS (USING ANY SOFTWARE)

(i) Solution of Cauchy problem for first order PDE.

(ii) Finding the characteristics for the first order PDE.

(iii) Plot the integral surfaces of a given first order PDE with initial data.

(iv) Solution of wave equation $\frac{\partial^2 u}{\partial t^2} - c\frac{\partial^2 u}{\partial x^2} = 0$ for the following associated conditions

- (a) $u(x, 0) = ¢(x)$, u_t{ $(x, 0) = \psi(x)$, $x \in R$, $t>0$
- *(b)* $u(x,0)=¢(X)$, u_t{ $(x, 0) = \psi(x)$, $u(0,t)=0$, $x\in (0,\infty)$, $t>0$
- (c) $u(x, 0)=¢(x)$, u_t { $(x, 0) = \psi(x)$, $u_x (0,t) = 0$, $x \in(O,\infty)$, $t>0$
- (d) $u(x, 0) = ¢(x)$, $ut\ (x, 0) = \psi(x)$, $u(0,t) = 0$, $u(l,t) = 0$, $0<x<l$, $t>0$

(v) Solution of wave equation $\frac{\partial u}{\partial t} - \kappa\frac{\partial^2 u}{\partial x^2} = 0$ for the following associated conditions

- (a) $u(x, 0) =\phi(x)$, $u(0,t) = a$, $u(l, t) = b$, $0 <x <l$, $t>0$
- (b) $u(x, 0) =\phi(x)$, $x \in R$, $0 <t <T$
- *(c)* $u(x,0)=\phi(x)$, $u(0,t)=a$, $x \in (0,\infty)$, $t \geq 0$

BOOKS RECOMMENDED :

1. Tyn Myint-U and Lokenath Debnath, *Linear Partial Differential Equations for Scientists and Engineers,* 4th edition, Birkhauser, Indian reprint, 2014.
2. S.L. Ross, *Differential equations,* 3rd Ed., John Wiley and Sons, India,

BOOK FOR REFERENCES:

1. J Sinha Roy and S Padhy:A course of Ordinary and Partial differential equation Kalyani Publishers,New Delhi,
2. Martha L Abell, James P Braselton, *Differential equations with MATHEMATICA,* 3rd Ed., Elsevier Academic Press, 2004.
3. Robert C. McOwen: Partial Differential Equations, Pearson Education Inc.
4. T Amarnath: An Elementary Course in Partial Differential Equations, Narosa Publications.

CORE PAPER-VIII

NUMERICAL METHODS AND SCIENTIFIC COMPUTING

Use of Scientific Calculator is allowed.

Objective: Calculation of error and approximation is a necessity in all real life, industrial and scientific computing. The objective of this course is to acquaint students with various numerical methods of finding solution of different type of problems, which arises in different branches of science such as locating roots of equations, finding solution of systems of linear equations and differential equations, interpolation, differentiation, evaluating integration.

Expected Outcome: Students can handle physical problems to find an approximated solution. After getting trained a student can opt for advance courses in Numerical analysis in higher mathematics. Use of good mathematical software will help in getting the accuracy one need from the computer and can assess the reliability of the numerical results, and determine the effect of round off error or loss of significance.

UNIT-I

Rate of convergence, Algorithms, Errors: Relative, Absolute, Round off, Truncation. Approximations in Scientific computing, Error propagation and amplification, conditioning, stability and accuracy, computer arithmetic mathematical software and libraries, visualisation, Numerical solution of non-linear equations: Bisection method, Regula-Falsi method, Secant method, Newton-Raphson method, Fixed-point Iteration method.

UNIT-II

Rate of convergence of the above methods. System of linear algebraic equations: Gaussian Elimination and Gauss Jordan methods. Gauss Jacobi method, Gauss Seidel method and their convergence analysis. Computing eigen-values and eigenvectors

UNIT-III

Polynomial interpolation: Existence uniqueness of interpolating polynomials.Lagrange and Newtons divided difference interpolation, Error in interpolation, Central difference & averaging operators, Gauss-forward and backward difference interpolation. Hermite and Spline interpolation, piecewise polynomial interpolation.

UNIT-IV

Numerical Integration: Some simple quadrature rules, Newton-Cotes rules, Trapezoidal rule, Simpsons rule, Simpsons *3/8th* rule, Numerical differentiation and integration, Chebyshev differentiation and FFT, Richard-son extrapolation.

PRACTICAL/LAB WORK TO BE PERFORMED ON A COMPUTER:

Use of computer aided software (CAS), for example *Matlab / Mathematica / Maple / Maxima* etc., for developing the following Numerical programs:

(i) Calculate the sum $1/1 + 1/2 + 1/3 + 1/4 +$---------- $+ 1/N$.

(ii) To find the absolute value of an integer.

(iii) Enter- 100 integers into an array and sort them in an ascending' order.

(iv) Any two of the following

- (a) Bisection Method
- (b) Newton Raphson Method
- (c) Secant Method
- (d) Regular Falsi Method
- (v) Gauss-Jacobi Method
- (vi) SOR Method or Gauss-Siedel Method
- (vii) Lagrange Interpolation or Newton Interpolation

(viii) Simpson's rule.

Note: For any of the CAS *Matlab / Mathematica / Maple / Maxima* etc., Data types-simple data types, floating data types, character data types, arithmetic operators and operator precedence, variables and constant declarations, expression, input/output, relational operators, logical operators and logical expressions, control statements and loop statements, Arrays should be introduced to the students.

BOOKS RECOMMENDED:

1. M. K. Jain, S. R. K. Iyengar and R. K. Jain, *Numerical Methods for Scientific and Engineering Computation,* New age International Publisher, India,
2. Michael Heath: Scientific Computing : An introductory Survey.

BOOK FOR REFERENCES:

1. B. Bradie, *A Friendly Introduction to Numerical Analysis,* Pearson Education, India, 2007.
2. Kendall E. Atkinson: An Introduction to Numerical Analysis
3. C. F. Gerald and P. O. Wheatley, *App.ied Numerical Analysis,* Pearson Education, India, 7th Edition, 2008
4. S. D. Conte & S. de Boor: Elementary Numerical Analysis: An Algorithmic Approach.

CORE PAPER-IX
TOPOLOGY OF METRIC SPACES

Objective: This is an introductory course in topology of metric spaces. The objective of this course is to impart knowledge on open sets, closed sets, continuous functions, connectedness and compactness in metric spaces.

Expected Outcomes: On successful completion of the course students will learn to work with abstract topological spaces. This is a foundation course for all analysis courses in future.

UNIT-I

Metric spaces, sequences in metric spaces, Cauchy sequences, complete metric spaces, open and closed balls, neighborhood, open set, interior of a set, limit point of a set, closed set, diameter of a set, Cantor's theorem,

UNIT-II

Subspaces, Countability Axioms and Separability, Baire's Category theorem

UNIT-III

Continuity: Continuous mappings, Extension theorems, Real and Complex valued Continuous functions, Uniform continuity, Homeomorphism, Equivalent metrics and isometry, uniform convergence of sequences of functions.

UNIT-IV

Contraction mappings and applications, connectedness, Local connectedness, Bounded sets and compactness, other characterization of compactness, continuous functions on compact spaces,

BOOKS RECOMMENDED:

1. Satish Shirali & Harikishan L. Vasudeva, *Metric Spaces*, Springer Verlag London (2006) (First Indian Reprint 2009)

BOOK FOR REFERENCES:

1. S. Kumaresan, *Topology of Metric Spaces*, Narosa Publishing House, Second Edition 2011.

CORE PAPER-X
RING THEORY

Objective: This is a second course in modern algebra which deals with ring theory. Some basics of ring theory like rings, subrings, ideals, ring homomorphisms and their properties and. This course is an integral part of any course on Modern algebra the ohers being Group theory and Field Theory.

Expected Outcomes: After completing this course, this will help students to continue more courses in advanced Ring theory modules, Galois groups.

UNIT-I

Definition and examples of rings, properties of rings, subrings, integral domains and fields, characteristic of a ring, Ideals, ideal generated by a subset of a ring, factor rings, operations on ideals.

UNIT-II

Prime and maximal ideals. Ring homomorphisms, properties of ring homomorphisms, Isomorphism theorems I, II and III, field of quotients.

UNIT-III

Polynomial rings over commutative rings, division algorithm and consequences, principal ideal domains, factorization of polynomials, reducibility tests, irreducibility tests, Eisenstein criterion, Unique factorization in Z[x].

UNIT-IV

Divisibility in integral domains, irreducibles, primes, unique factorization domains, Euclidean domains.

BOOKS RECOMMENDED:

1. Joseph A. Gallian, *Contemporary Abstract Algebra* (4th Edition), Narosa Publishing House, New Delhi.
2. John B. Fraleigh, *A First Course in Abstract Algebra,* 7th Ed., Pearson, 2002.

BOOK FOR REFERENCES:

1. M. *Artin, Abstract Algebra,* 2nd Ed., Pearson, 2011.
2. Joseph 1. Rotman, *An Introduction to the Theory of Groups,* 4th Ed., Springer Verlag, 1995.
3. I. N. Herstein, *Topics in Algebra,* Wiley Eastern Limited, India, 1975.

STATE MODEL SYLLABUS FOR UNDER GRADUATE COURSE IN SKILL ENHANCEMENT COURSE (II)

(Bachelor of Arts/Sc/Com Examination)

UNDER

CHOICE BASED CREDIT SYSTEM

FOREWARD

The Higher Education system has undergone a paradigm shift in Odisha with the introduction of Choice Based Credit System (CBCS) in the academic year 2015-16 as per the University Grant Commission regulations. Initially it was adopted in all Autonomous colleges and from 2016-17, in all the colleges of Odisha. CBCS offers students the liberty to choose from list of available courses under the domains of Ability Enhancement, Skill Enhancement and General Elective. This book on Quantitative and Logical Thinking aims to engage the students more creatively to improve their critical thinking skills. This paper will be taught under Skill Enhancement Compulsory Course (SECC).

The main intent of this paper is to strengthen the quantitative & logical thinking of Under Graduate students, majority of who are set to enter the job market with high hopes. Needless to say, a good command over Quantitative Aptitude and Logical Thinking is one skill which various companies expect from their prospective employees. The course content is developed with the help of faculties from Ravenshaw University, Rama Devi University and other experienced Mathematics faculties keeping in mind the diverse background of students of Odisha. We would like to acknowledge their vital contribution and members of the World Bank project in Higher Education for the development of this book. We hope the students find merit in using this book not just as a course study material but as a life time companion in improving his / her critical thinking skills. Any suggestions for improving the content are most welcome. The same can be emailed to oshec.hed@gmail.com

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Table of Contents

QUANTITATIVE AND LOGICAL THINKING

I. QUANTITATIVE APTITUDE & DATA INTERPRETATION

Unit – 1: Whole numbers, Integers, Rational and irrational numbers, Fractions, Square roots and Cube roots, Surds and Indices, Problems on Numbers, Divisibility

Definitions:

- Whole numbers:

All positive numbers including 0 are called whole numbers.

For Example - 0, 1, 2, 3 ...

- Prime numbers:

A number that is divisible only by itself and 1 is called a prime number.

For Example – 2, 3, 5, 7, …..

  - ❖ Prime numbers are whole numbers.
  - ❖ The smallest prime number is 2.

- Co-prime / Relatively prime/ Mutually prime:

Two numbers *a* and *b* (not both necessarily prime) are said to be co-prime, relatively prime or mutually prime if the only positive integer that divides both of them is 1.

For Example- 15 and 22 are co-prime because the only common divisor is 1.

- Integers:

All the positive and negative numbers including 0 are called integers.

For Example- -3, -2, -1, 0, 1, 2, 3 ...

- Rational numbers:

The set of numbers which can be written in the form of (p/q) are called rational numbers .

For Example - 115/4, 0, 26/5 , -22/9...

- <u>Irrational numbers:</u>

The set of numbers which cannot be written in the form of (p/q) are called irrational numbers.

For Example - $\pi, \sqrt{2}, \sqrt[3]{3}$…

- <u>Real numbers:</u>

Real numbers contains the set of Whole numbers, integers, rational and irrational number.

For Example: 1, -2, 0, $\pi, \sqrt{2}$….

Basic formulae

1. $(a+b)^2 = a^2 + b^2 + 2ab$
2. $(a-b)^2 = a^2 + b^2 - 2ab$
3. $(a+b)^2 - (a-b)^2 = 4ab$
4. $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$
5. $(a^2-b^2) = (a+b)(a-b)$
6. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$
7. $(a^3+b^3) = (a+b)(a^2-ab+b^2)$
8. $(a^3-b^3) = (a-b)(a^2+ab+b^2)$
9. $(a^3+b^3+c^3-3abc) = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Solved examples:

1. If one-third of one-fourth of a number is 15, then three-tenth of that number is:

Solution:

Let the number be x.

Then, $\frac{1}{3}$ of $\frac{1}{4}$ of $x = 15 \Rightarrow x = 15 \times 12 = 180$.

So, required number = $\left(\frac{3}{10} \times 180\right) = 54$.

2. The sum of two numbers is 25 and their difference is 13. Find their product.

Solution:

Let the numbers be x and y.

Then, $x + y = 25$ and $x - y = 13$.

$4xy = (x + y)^2 - (x - y)^2$

$= (25)^2 - (13)^2$

$= (625 - 169)$

$= 456$

$\therefore \quad xy = 114.$

3. The difference between a two-digit number and the number obtained by interchanging the positions of its digits is 36. What is the difference between the two digits of that number?

Solution:

Let the ten's digit be x and unit's digit be y.
Then, $(10x + y) - (10y + x) = 36$
$\Rightarrow 9(x - y) = 36$
$\Rightarrow x - y = 4.$

4. The difference between a two-digit number and the number obtained by interchanging the ts is 36. What are the digits of the number if the ratio between the digits of the number is 1 : 2 ?

Solution:

Since the number is greater than the number obtained on reversing the digits, so the ten's digit is greater than the unit's digit.

Let ten's and unit's digits be $2x$ and x respectively.
Then, $([10 \text{ x } 2x] + x) - (10x + 2x) = 36$
$\Rightarrow 9x = 36$
$\Rightarrow x = 4.$
$\therefore$ One digit is 4 and the other digit is $2\times 4 = 8$

5. The product of two numbers is 18 and the sum of their squares is 45. The sum of the numbers is:

Solution:

Let the numbers be x and y.
Then, $xy = 18$ and $x^2 + y^2 = 45$.
$\therefore \ (x + y)^2 = x^2 + y^2 + 2xy = 45 + (2 \times 18) = 81$
$\therefore \ x + y = \sqrt{81} = 9.$

Divisibility rules

Divisibility by 2:

A number is divisible by 2 if the last digit is even. i.e., if the last digit is 0 or 2 or 4 or 6 or 8.
Ex: 454 is divisible by 2 , 455 is not divisible by 2.

Divisibility by 3:

A number is divisible by 3 if the sum of the digits is divisible by 3.
Ex: 459 is divisible by 3 as the sum of the digits, 4+5+9=18 is divisible by 3.

Divisibility by 4:

A number is divisible by 4 if the number formed by the last two digits is divisible by 4.
Ex: 324 is divisible by 4 as 24 is divisible by 4.

Divisibility by 5:

A number is divisible by 5 if the last digit is either 0 or 5.

Ex: 555 is divisible by 5

Divisibility by 6:

A number is divisible by 6 if it is divisible by both 2 and 3.

Ex: 528 is divisible by 6 as 528 is divisible by both 2 & 3.

Divisibility by 8:

A number is divisible by 8 if the number formed by the last three digits is divisible by 8.

Ex: 8168 is divisible by 8 as 168 is divisible by 8.

Divisibility by 9:

A number is divisible by 9 if the sum of the digits is divisible by 9.

Ex: 981 is divisible by 9 as the number formed by the sum of the digits i.e. 18(9+8+1=18) is divisible by 9.

Divisibility by 10:

A number is divisible by 10 if the last digit is 0.

Ex: 100 is divisible by 10.

Divisibility by 11:

To find out if a number is divisible by 11, find the sum of the odd numbered digits and the sum of the even numbered digits.

Now subtract the lower number obtained from the bigger number obtained.

If the number we get is 0 or divisible by 11, the original number is also divisible by 11.

Ex: 121 is divisible by 11. (Sum of the digits in the even place is 2 & sum of the digits in the odd places is 1+1=2.Now 2-2 =0 is divisible by 11.)

Solved examples:

1. Find the least value of * for which the number 8550*1 is divisible by 3.

Solution:

Let the required number be a.
Now 8+5+5+0+a+1 = 19+a

Hence the least number is 2.

2. Find the least value of * for which the number 13*1 is divisible by 11.

Solution:

Let the required number be x.
Now sum of digits at odd places –sum of digits at even places
= (1+3)-(1+x)
= (3-x), should be divisible by 11.
Hence the least value of x is 3.

3. Is 7248 divisible (i) by 4, (ii) by 2 and (iii) by 8?

Solution:

(i) The number 7248 has 48 on its extreme right side which is exactly divisible by 4. When we divide 48 by 4 we get 12.

Therefore, 7248 is divisible by 4.

(ii) The number 7248 has 8 on its unit place which is an even number so, 7248 is divisible by 2.

(iii) 7248 is divisible by 8 as 7248 has 248 at its hundred place, tens place and unit place which is exactly divisible by 8.

4. A number is divisible by 4 and 12. Is it necessary that it will be divisible by 48? Give an example in support of you answer.

Solution:

$48 = 4 \times 12$ but 4 and 12 are not co-prime.

Therefore, it is not necessary that the number will be divisible by 48.

Let us consider the number 72 for an example

$72 \div 4 = 18$, so 72 is divisible by 4.

$72 \div 12 = 6$, so 72 is divisible by 12.

But 72 is not divisible by 48.

5. Without actual division, find if 235932 is divisible (i) by 4 and (ii) 8.

Solution:

(i)The number formed by the last two digits on the extreme right side of 235932 is 32

$32 \div 4 = 8$, i.e. 32 is divisible by 4.

Therefore, 235932 is divisible by 4.

(ii) The number formed by the last three digits on the extreme right side of 235932 is 932

But 932 is not divisible by 8. Therefore, 235932 is not divisible by 8.

Square roots

Steps of Long Division Method for Finding Square Roots:

Step I: Group the digits in pairs, starting with the digit in the units place.

Step II: Think of the largest number whose square is equal to or just less than the first group. Take this number as the divisor and also as the quotient.

Step III: Subtract the product of the divisor and the quotient from the first group and bring down the next group to the right of the remainder. This becomes the new dividend.

Step IV: Now, the new divisor is obtained by taking two times the quotient and annexing with it a suitable digit which is also taken as the next digit of the quotient, chosen in such a way that the product of the new divisor and this digit is equal to or just less than the new dividend.

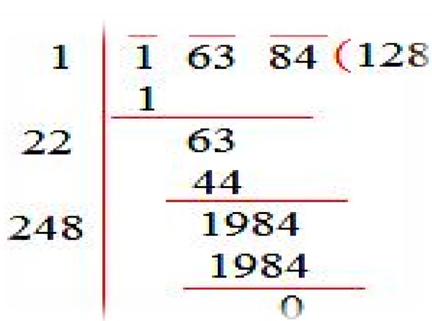
Step V: Repeat steps (2), (3) and (4) till all the groups have been taken up. Now, the quotient so obtained is the required square root of the given number.

Solved examples:

1. Find out √16384.

Solution:

Marking group and using the long-division method,

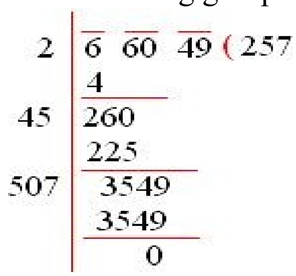


Therefore, √16384 = 128.

2. Find out √66049.

Solution:

Marking group and using the long-division method,



Therefore, √66049 = 257

Surds & Indices

Definitions:

- Index

An **index** (plural: **indices**) is the power, or exponent, of a number. E.g a^5 has an **index** of 5.

- Surd

A **surd** is an irrational number that can be expressed with roots.

E.g. Let *a* be rational number and *n* be a positive integer such that $(a)^{(1/n)} = \sqrt[n]{a}$, Then a is called a surd of order n.

Laws of Indices:

1. $a^m \times a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $(ab)^n = a^n b^n$

4. $\frac{a^m}{a^n} = a^{m-n}$

5. $a^0 = 1$

6. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

7. $a^{-n} = \frac{1}{a^n}$

Laws of Surds:

1. $\sqrt[n]{a} = a^{(1/n)}$
2. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
3. $\sqrt[n]{\frac{a}{b}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}$

Solved examples:

1. Divide 12 by $3\sqrt{2}$.

Solution:

$\frac{12}{3\sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{4\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = 2\sqrt{2}$.

2. Simplify$(125)^{-2/3}$.

Solution:

$(125)^{-2/3}$

$= (5 \times 5 \times 5)^{-2/3} = (5)^{3\times\frac{-2}{3}}$

$= 5^{-2} = \frac{1}{5^2}$

$= \frac{1}{25}$

3. $15^4 \times 15^s = 15^6$ Find the value of x.

Solution:

$15^4 \times 15^s = 15^6$

$\Rightarrow$ $15^s = \frac{15^6}{15^4}$

$\Rightarrow$ $15^s = 15^2$

$\Rightarrow$ $x = 2$

4. If $2^a = 64$, then find the value of 2^{a-3}.

Solution:

We have $2^a = 64$ (Since the LHS contains a power of 2 so the RHS must be expressed in terms of some power of 2)

$\Rightarrow$ $2^a = 2^6$

$\Rightarrow$ $a = 6$

$\therefore$ $2^{a-3} = 2^{6-3} = 2^3 = 8$.

Unit -2: Basic concepts, Different formulae of Percentage, Profit and Loss, Discount, Simple interest, Ratio and Proportion, Mixture

Definitions:

Percentage is a fraction whose denominator is always 100. x percentage is represented by

$$x\,\% = \frac{s}{100}$$

Example - $25\,\% = \frac{25}{100} = \frac{1}{4}$

Formulae:

1. To express $\frac{s}{y}$ as a percentage

   We know that $\frac{s}{y} = (\frac{s}{y} \times 100)\,\%$

   E.g.- $\frac{1}{4} = (\frac{1}{4} \times 100)\,\% = 25\%$

2. If A is R % more than B , then B is less than A by $(\frac{R}{R+100} \times 100)\,\%$.

3. If A is R % less than B , then B is more than A by $(\frac{R}{100-R} \times 100)\,\%$.

4. If the price of a commodity increases by R % , then reduction in consumption as not to increase the expenditure is $(\frac{R}{R+100} \times 100)\,\%$.

5. If the price of a commodity decreases by R % , then increase in consumption as not to decrease the expenditure is $(\frac{R}{100-R} \times 100)\,\%$.

6. Results on Population : Let the population of a town be P now and suppose increases at the rate of R % per annum , then :

   i) Population after n years $= P\,(1 + \frac{R}{100})^n$.

ii) Population n years ago $= \dfrac{P}{\left(1+\frac{R}{100}\right)^n}$

7. Results on Depreciation: Let the present value of a machine be P . Suppose it depreciates at the rate of R % per annum , then :

i) Value of machine after n years = $P\left(1-\frac{R}{100}\right)^n$

ii) Value of machine n years ago $= \dfrac{P}{\left(1-\frac{R}{100}\right)^n}$

8. To remember :

$\frac{1}{2} = 50\ \%$

$\frac{1}{4} = 25\ \%$

$\frac{1}{6} = 16\frac{2}{3}\ \%$

$\frac{1}{8} = 12\frac{1}{2}\ \%$

$\frac{1}{10} = 10\ \%$

$\frac{1}{12} = 8\frac{1}{3}\ \%$

$\frac{1}{3} = 33\frac{1}{3}\ \%$

$\frac{1}{5} = 20\ \%$

$\frac{1}{7} = 14\frac{2}{7}\ \%$

$\frac{1}{9} = 11\frac{1}{9}\ \%$

$\frac{1}{11} = 9\frac{1}{11}\ \%$

$\frac{1}{13} = 7\frac{9}{13}\ \%$

Solved examples:

1. Find 8 % of 625.

Solution : $\frac{8}{100} \times 625 = 50.$

2. Ram's salary is increased from Rs. 24,000 to Rs. 30,000. Find the increased % .

Solution :

Increase in salary = Rs.30,000 – Rs.24,000 = Rs. 6000

% Increase = $\frac{6000}{24000} \times 100 = 25\ \%$.

3. In an election, candidate A got 75% of the total valid votes. If 15% of the total votes were declared invalid and the total numbers of votes is 560000, find the number of valid vote polled in favor of candidate.

Solution:

Given that 15% of the total votes were invalid. So we have, Total number of valid votes = 85 % of 560000

$$= \frac{85}{100} \times 560000$$

$$= \frac{47600000}{100}$$

$$= 476000$$

Percentage of votes polled in favor of candidate A = 75 %

Therefore, the number of valid votes polled in favors of candidate A = 75 % of 476000

$$= \frac{75}{100} \times 476000$$

$$= \frac{35700000}{100}$$

$$= 357000$$

4. Rama had Rs.2100 left after spending 30 % of the money he took for shopping. How much money did he take along with him?

Solution:

Let the money Rama took along with him be 100%.

Given that he spent 30 % of the money for shopping. So money left with him is 70% of the money.

But money left with him = Rs. 2100

Therefore 70%= Rs. 2100

$\therefore$ 100% = Rs 2100$\times \frac{100}{70}$

$\therefore$ 100% = Rs 3000

Therefore, the money he took for shopping is Rs 3000.

5. A shopkeeper bought 600 oranges and 400 bananas. He found 15% of oranges and 8% of bananas were rotten. Find the percentage of fruits in good condition.

Solution:

Total number of fruits shopkeeper bought = 600 + 400 = 1000

Given that 15% of oranges and 8% of bananas were rotten. So he had 85% of oranges & 92 % of bananas in good condition.

Number of oranges in good condition = 85% of 600

$$= \frac{85}{100} \times 600$$

$$= 510$$

Number of bananas in good condition= 92 % of 400

$$= \frac{92}{100} \times 400$$

$$=368$$

Therefore Number of fruits in good condition = 510+ 368 = 878

Therefore Percentage of fruits in good condition = $\left(\frac{878}{1000} \times 100\right)\%$

$$= 87.8\%$$

PROFIT & LOSS

Definitions:

- Cost Price: The price at which an article is purchased is called its cost price and it is denoted by CP.
- Selling Price: The price at which an article is sold is called its selling price and denoted by SP.
- Profit (P) :If SP is greater than CP, then seller is said to have a profit.
- Loss (L) : If SP is less than CP , then seller is said to have a loss.
- Marked price: MRP of an article is known as marked price or labeled price and denoted by MP.
- Discount : Discount is a percentage of the MP.

  * Profit and loss are always counted on CP.

* Discount is always carried on MP.

Formulae:

1. $P = SP - CP$

2. $L = CP - SP$

3. $P\% = \frac{P}{CP} \times 100$

4. $L\% = \frac{L}{CP} \times 100$

5. $SP = \frac{100+P\%}{100} \times CP$

6. $SP = \frac{100-L\%}{100} \times CP$

7. $CP = \frac{100}{100+P\%} \times SP$

8. $CP = \frac{100}{100-L\%} \times SP$

9. Discount (D) = MP - SP

10. $\text{Discount}\ \% = \frac{\text{Diccount}}{MP} \times 100$

11. $SP = \frac{100-D\%}{100} \times MP$

Solved examples:

1. A person purchased an article for Rs. 80 and sold it for Rs.120. Find his % of profit.

Solution :

CP of the article = Rs. 80

SP of the article = Rs. 120

Profit = SP – CP

$= \text{Rs. } 120 - \text{Rs. } 80 = \text{Rs. } 40$

Profit % $= \frac{40}{80} \times 100 = 50\ \%$.

2. By selling a fan for Rs 649, Anil earns a profit of 18%. Find its cost price.

Solution:

S.P. of the fan = Rs 649, profit = 18%

Therefore, Rs 649 = $(1 + \frac{18}{100})$ of C.P.

$\therefore$ Rs 649 = $\frac{118}{100}$ of C.P.

$\therefore$ C.P. = Rs $(649 \times \frac{100}{118})$ = Rs 550

Hence the cost price of the fan = Rs 550.

3. Sammy sold his dining table set at a loss of 20%. If he had sold it for Rs. 800 more, he would have received a profit of 5%. Find the cost price.

Solution :

Let the cost price be Rs. 100

So when C.P = 100 , loss of 20% means

S.P = 100 – 20 = 80

Profit of 5% means S.P = 100 + 5 = 105

The difference of two S.P = 105 - 80 = 25

If the difference is 25, C.P = Rs100

If the difference is Rs 800 , C.P = (100 / 25) x 800

C.P = Rs 3200

4. The cost of 11 pencils is equal to the selling price of 10 pencils. Find the loss or profit percent, whatever may be the cost of 1 pencil.

Solution:

The cost price of 11 pencils = S.P of 10 pencils

Let C.P of 1 pencil is Rs.1.

C.P of 10 pencils = Rs. 10

S.P of 10 pencils = C.P of 11 pencils = Rs. 11

Profit on 10 pencils = 11 – 10 = Rs. 1

Profit % = $(\frac{1}{10}) \times 100 = 10$ %.

5. A person sold an article at a profit of 12 %. If he had sold it Rs. 4 more, he would have gained 20 %. What is the cost price?

Solution:

Let the CP of an article be Rs x. Then,

112 % of x + 4 = 120 % of x

$\Rightarrow$ 120 % of x - 112 % of x = 4

$\Rightarrow$ 8 % of x = 4

$\Rightarrow \frac{8}{100} \times x = 4$

$\Rightarrow x = 4 \times \frac{100}{8} = 50.$

DISCOUNT

Solved examples:

1. If the Marked Price of an article is Rs 1000, then what is the Selling Price at a discount rate of 20%?

Solution:

Given MP of an article= Rs. 1000

D%=20

So SP= $1000 \times \frac{80}{100} = 800.$

2. Find the selling price of an article after two successive discount 10% & 20% if the marked price is Rs. 2500.

Solution:

Given MP of an article = Rs. 2500

$$SP = \frac{100 - D\,\%}{100} \times MP$$

So after first discount 10%, SP = $\frac{100-10}{100} \times 2500 = \text{Rs. } 2250$

Now the new MP is = Rs. 2250

After 20% successive discount SP = $\frac{100-20}{100} \times 2250 = \text{Rs. } 1800$

3. If the marked price of an article is 13% more than CP and a shopkeeper allows a discount of 10%. Find the profit/ loss percentage?

Solution:

Let CP of the article be Rs. 100.

Then MP = 113% of 100 = 113

Discount = 10%

SP $= 113 \times \frac{90}{100}$

$= 101.7$

P = SP $-$ CP $= 101.7 - 100 = 1.7$

% P = $\frac{P}{CP} \times 100 = \frac{1.7}{100} \times 100$ = 1.7 %

4. After getting two successive discounts, a shirt with MRP Rs. 500 is available at Rs.420. If the first discount is 12.5% then find out the percentage of second discount.

Solution:

Let the second discount be x %.

Thus, (100 – x) % of 87.5% of 500 = 420

$\Rightarrow \frac{100-s}{100} \times \frac{87.5}{100} \times 500 = 420$

$\Rightarrow$ 100 - x = 96

$\Rightarrow$ x = 4 %.

5. At what % above the CP must an article be marked so as to gain 17 % allowing 10 % discount?

Solution:

Let CP of the article be 100.

Then SP = 117

Let MP of the article be x. Now,

90 % of x = 117

$\Rightarrow \frac{90}{100} \times x = 117$

$\Rightarrow x = 117 \times \frac{100}{90} = 130.$

So, MP = 30 % above CP.

SIMPLE INTEREST

Definition :

- Principal (P): The money borrowed or lent out for a certain period is called principal.
- Interest (I): Extra money paid for using other 's money is called interest.
- Simple Interest (S.I) : If the interest on the money borrowed is paid uniformly , then it is called simple interest.

Formulae :

1. $S.I = \frac{P \times R \times T}{100}$

Where , P = principal

R = rate percent per annum

T = time period (Number of years)

2. Amount (A) = P + S.I

Conversions

1. Case I : If S.I , R and T are known ,

   $P = \frac{S.I \times 100}{R \times T}$

2. Case II : If S.I , P and T are known ,

   $R = \frac{S.I \times 100}{P \times T}$

3. Case III : If S.I , P and R are known ,

   $T = \frac{S.I \times 100}{P \times R}$

Solved examples:

1. Find S.I on Rs 2000 at the rate of interest 10 % p.a. for 2 years .

Solution:

$S.I = \frac{P \times R \times T}{100}$

$= \frac{2000 \times 10 \times 2}{100}$

= Rs. 400

2. Find S.I and the amount on Rs. 4000 at a rate of interest 5 % for 6 months.

Solution:

Here, P = Rs. 4000 , R = 5 % , T = 6 months = $\frac{6}{12}$ years = $\frac{1}{2}$ year

$S.I = \frac{P \times R \times T}{100}$

$= \frac{4000 \times 5 \times 1}{100 \times 2}$ = Rs. 100

Amount = P + S.I

= Rs. 4000 + Rs. 100 = Rs. 4100

3. In what time will Rs. 3100 amount to Rs. 6200 at 4 % p.a ?

Solution :

Here , P = Rs. 3100 , R = 4 % , A = Rs. 6200

So, Interest = Rs. 6200 – Rs. 3100 = Rs. 3100

$$T = \frac{3100 \times 100}{3100 \times 4} = 25 \text{ years}$$

4. A sum of money put at S.I doubles itself in 8 years . In how many years it will become five times ?

Solution :

Let Principal be P. Then, Amount = 2 P

$\Rightarrow$ S.I = 2P - P = P

Using, S.I = $\frac{P \times R \times T}{100}$

$\Rightarrow$ $P = \frac{P \times R \times 8}{100}$

$\Rightarrow$ $R = \frac{100}{8}$

Again Principal = P

$\Rightarrow$ Amount = 5 P

$\Rightarrow$ S.I = 5 P – P = 4 P

Again using, S.I = $\frac{P \times R \times T}{100}$

$\Rightarrow$ $4P = \frac{P \times \frac{100}{8} \times T}{100}$

$\Rightarrow$ T = 4 × 8 = 32 years.

5. If S.I is $\frac{1}{4}$th of the principal and the number of years is equal to rate of interest , then find the rate percent p.a.?

Solution :

Let Principal = P

$\Rightarrow$ S.I = $\frac{1}{4}$P

Let Rate of interest be x .

As per the question, Time = rate of interest = x .

$$S.I = \frac{P \times R \times T}{100}$$

$$\Rightarrow \frac{1}{4}P = \frac{P \times s \times s}{100}$$

$$\Rightarrow x^2 = \frac{100}{4} = 25$$

$$\Rightarrow x = 5$$

RATIO AND PROPORTION

Definition:

- Ratio: The ratio of two quantities a and b in the same units is the fraction $\frac{a}{b}$ and we write it as a : b. In the ratio a : b , a is the first term or antecedent and b is the second term or consequent.

  Example - In the ratio 5 : 9 , 5 is the antecedent and 9 is the consequent.

- Proportion: The equality of two ratios is called proportion.

  If a : b = c : d , then **a, b , c** and **d** are in proportion and can also be written as a : b : : c : d

Solved examples:

1. Divide 240 into two parts in the ratio 2 : 3.

Solution –

Let the first part be 2x and the second part be 3x.

Now, 2x + 3x = 240

$\Rightarrow$ 5x = 240

$\Rightarrow$ x = 48

So 2x = 2 × 48 = 96 and 3x = 3 × 48 = 144.

2. Find three numbers in the ratio 1 : 3 : 5 so that the sum of their squares is equal to 315.

Solution –

Let the numbers be x , 3x , 5x.

Now, we have

$x^2 + (3x)^2 + (5x)^2 = 315$

$\Rightarrow x^2 + 9x^2 + 25x^2 = 315$

$\Rightarrow 35x^2 = 315$

$\Rightarrow x^2 = 9$

$\Rightarrow x = 3$

$\Rightarrow 3x = 9$ & $5x = 15$

$\therefore$ The required numbers are 3, 9 and 15.

3. A mixture contains milk and water in the ratio 5 : 4. If 5 litres of water is added to the mixture, the ratio becomes 5 : 6 . Find the quantity of milk in the given mixture.

Solution –

Let the quantity of milk and water be 5x and 4x litres respectively. Then, $\frac{5s}{4s+5} = \frac{5}{6}$

$\Rightarrow 30x = 20x + 25$

$\Rightarrow 10x = 25$

$\Rightarrow x = 2.5$ litres

$\Rightarrow 5x = 5 \times 2.5 = 12.5$ litres

Thus, the quantity of milk in the given mixture is 12.5 litres.

4. The sides of a triangle are in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ and its perimeter is 78 cm. Find the length of the sides of the triangle.

Solution –

Let the sides of the triangle be $a = \frac{1}{2}x$, $b = \frac{1}{3}x$, $c = \frac{1}{4}x$

Given, Perimeter of the triangle is 78 cm.

$\Rightarrow \frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x = 78$

$\Rightarrow \frac{13}{12}x = 78$

$\Rightarrow x = 72$ cm

$\Rightarrow a = 36$ cm , $b = 24$ cm and $c = 18$ cm .

MIXTURE & ALLIGATION

Definition:

- Alligation**:** It is a rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of desired price.
- CP of the mixture: It is the cost price of a unit quantity of mixture.

- It is a modified form of finding the weighted average. If 2 ingredients are mixed in a ratio and the cost price of the unit quantity of the mixture, called the cost of mixture per kg is given then,

  $\Rightarrow Q_c \times (m - c) = Q_d \times (d - m)$

- Here 'd' is the cost of dearer ingredient, 'm' is the cost of mixture per kg and 'c' is the cost of cheaper ingredient. Q_c & Q_d are the quantity of the cheaper and dearer ingredient respectively.

Solved examples:

1. The cost of Type 1 material is Rs. 15 per kg and Type 2 material is Rs.20 per kg. If both Type 1 and Type 2 are mixed in the ratio of 2 : 3, then what is the price per kg of the mixed variety of material?

Solution –

Cost Price (CP) of Type 1 material is Rs. 15 per kg
Cost Price (CP) of Type 2 material is Rs. 20 per kg

Type 1 and Type 2 are mixed in the ratio of 2 : 3.

Hence Cost Price (CP) of the resultant mixture

$$= \frac{(15 \times 2) + (20 \times 3)}{(2+3)} = \frac{90}{5} = 18$$

Price per kg of the mixed variety of material = Rs.18.

2. A mixture of 30 litres of milk and water contains 30% of water. The new mixture is formed by adding 10 lit of water. What is the percentage of water in the new mixture?

Solution -

Quantity of water in the 30 litre mixture = $\frac{30}{100} \times 30$ = 9 litre
After adding 10 litre of water, quantity of water becomes 19 litre and total quantity becomes 40 litre.
Percentage of water = $\frac{19}{40} \times 100$ = 47.5%

3. Tea worth of Rs. 135/kg & Rs. 126/kg are mixed with a third variety in the ratio 1: 1 : 2. If the mixture is worth Rs. 153 per kg, the price of the third variety per kg will be ?

Solution –

Let the price of the third variety of tea be Rs. x.

Given that Tea worth of Rs. 135/kg , Rs. 126/kg &Rs. x/kg are mixed in the ratio 1: 1 : 2 and the mixture is worth Rs. 153 per kg

Now we have, $\frac{135 \times 1 + 126 \times 1 + s \times 2}{1+1+2} = 153$

$\Rightarrow \frac{135 + 126 + 2s}{4} = 153$

$\Rightarrow$ 261 + 2x = 153 × 4 = 612

$\Rightarrow$ 2x = 612 – 261 = 351

$\Rightarrow$ x = 175.50

Hence, price of the third variety = Rs.175.50 per kg.

4. A merchant has 1000 kg of sugar part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The Quantity sold at 18% profit is?

Solution –

By the rule of alligation:

$Q_c \times (m- c) = Q_d \times (d- m)$

$\Rightarrow \frac{Q_c}{Qd} = \frac{(d-m)}{(m-c)} = \frac{18-14}{14-8} = \frac{4}{6} = \frac{2}{3}$

So, ratio of cheaper quantity and dearer quantity = 2 : 3

∴ Quantity of dearer part = $\frac{3}{5} \times 1000$= 600 Kg

5. A mixture of 150 litres of wine and water contains 20% water. How much more water should be added so that water becomes 25% of the new mixture?

Solution –

Number of litres of water in 150 litres of the mixture = 20% of 150 = $\frac{20}{100} \times 150$ = 30 litres

Let us assume that another 'P' litre of water is added to the mixture to make water 25% of the new mixture. So, the total amount of water becomes (30 + P) and the total volume of the mixture becomes (150 + P)

Thus, (30 + P) = 25% of (150 + P)

$\Rightarrow 30 + P = \frac{25}{100} \times (150 + P)$

$\Rightarrow 30 + P = \frac{(150 + P)}{4}$

$\Rightarrow 120 + 4P = 150 + P$

$\Rightarrow$ 4P – P = 30
$\Rightarrow$ 3 P = 30

∴ We get P = 10 litres.

Unit- 3: Time and Work, Pipes and Cisterns, Basic concepts of Time, Distance and Speed ; relationship among them

- Work done is dependent on factors like number of persons working, number of days, number of hours working per day etc.,If M1 persons working D1 days can complete W1 amount of work and M2 persons working D_2 days can complete W_2 amount of work , then we have a general formula in the relationship of

$$\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}$$

$$\Rightarrow M_1D_1W_2 = M_2D_2W_1$$

- If we include the working hours (say, T_1 and T_2) for the two groups and efficiency (say, E_1 and E_2) of the persons in two groups , then the relationship is

$$\frac{M_1D_1T_1E_1}{W_1} = \frac{M_2D_2T_2E_2}{W_2}$$

$$\Rightarrow M_1D_1W_2T_1E_1 = M_2D_2W_1T_2E_2$$

Important Formulae

1. Work from days
If A can do a piece of work in n days, then A's 1 day's work = $\frac{1}{n}$

2. Days from Work
If A's 1 day's work = $\frac{1}{n}$, then A can finish the work in n days.

3. Ratio

a) If A is thrice as good a workman as B, then ,

i. Ratio of work done by A and B =3:1.

ii. Ratio of times taken by A and B to finish a work =1:3

b) If A is x times as good a workman as B, then he will take $\left(\frac{1}{s}\right)^{th}$ of the time by B to do the same work.

1. A and B can do a piece of work in 'x' days and 'y' days respectively, then working together, they will take $\left(\frac{sy}{s+y}\right)$ days to finish the work and in one day, they will finish $\left(\frac{s+y}{sy}\right)^{th}$ part of work.

Solved examples:

1. A can do piece of work in 30 days while B alone can do it in 40 days. In how many days can A and B working together do it?

Solution –

A′s one day’s work = $\frac{1}{30}$

B′ s one day’s work = $\frac{1}{40}$

(A+B)′s one day’s work = $\frac{1}{30} + \frac{1}{40} = \frac{4+3}{120} = \frac{7}{120}$

∴ Number of days required for A and B to finish the work = $\frac{1}{7/120} = \frac{120}{7} = 17\frac{1}{7}$ days.

This can also be calculated by using the formula as…

2. To complete a piece of work A and B take 8 days, B and C 12 days. A, B and C take 6 days. A and C will take **:**

Solution –

Given (A+B)'s one day’s work = $\frac{1}{8}$

(B+C)'s one day’s work = $\frac{1}{12}$

(A+B+C)'s 1 day's work = $\frac{1}{6}$

Work done by A, alone = (A+B+C)'s 1 day's work - (B+C)'s one day's work

$= \frac{1}{6} - \frac{1}{12}$

$= \frac{1}{12}$

Work done by C, alone = (A+B+C)'s 1 day's work - (A+B)'s one day's work

$= \frac{1}{6} - \frac{1}{8}$

$= \frac{1}{24}$

(A+C)'s one day's work = $\frac{1}{12} + \frac{1}{24} = \frac{1}{8}$

$\therefore$ (A+C) will take 8 days to complete the work together.

3. A and B can do a piece of work in 45 days and 40 days respectively. They began to do the work together but A leaves after some days and then B completed the remaining work in another 23 days. The number of days after which A left the work was?

Solution –

(A+B)'s 1 day's work = $\frac{1}{45} + \frac{1}{40} = \frac{17}{360}$

Work done by B in 23 days = $1 \times \frac{23}{40} = \frac{23}{40}$

Remaining work = $1 - \frac{23}{40} = \frac{17}{40}$

Now, $\frac{17}{360}$ work was done by (A+B) in 1 day.

$\frac{17}{40}$ Work was done by (A+B) in $1 \times \frac{360}{17} \times \frac{17}{40} = 9$ days

$\therefore$ A left after 9 days.

4. A and B undertake to do a piece of work for Rs 600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they can finish it in 3 days, Find the share of C in Rs?

Solution –

A's one day's work = $\frac{1}{6}$

B's one day's work = $\frac{1}{8}$

(A+ B + C)'s one day's work = $\frac{1}{3}$

$\therefore$ C's one day's work,= $\left(\frac{1}{3}\right) - \left(\frac{1}{6} + \frac{1}{8}\right)$

$= \frac{1}{24}$

Therefore, A : B : C = Ratio of their one day's work

$= \frac{1}{6} : \frac{1}{8} : \frac{1}{24}$

C's share for 3 days = $\frac{1}{24} \times 3 \times 600$ = Rs. 75

5. A can build up a structure in 8 days and B can break it in 3 days. A has worked for 4 days and then B joined to work with A for another 2 days only. In how many days will A alone build up the remaining part of the structure?

Solution –

A can build the structure in 8 days.

Fraction of structure built in a day by A = $\frac{1}{8}$

Similarly, fraction of structure broken by B in a day = $\frac{1}{3}$

Amount of work done by A in 4 days = $\frac{4}{8} = \frac{1}{2}$

Now, both A and B together for 2 days.

Amount of work done by A in 2 days = $\frac{2}{8} = \frac{1}{4}$

Amount of structure broken by B in 2 days = $\frac{2}{3}$

Fraction of structure built = $\left(\frac{1}{2} + \frac{1}{4}\right) - \frac{2}{3} = \frac{1}{12}$

Fraction of structure still to be built = $1 - \frac{1}{12} = \frac{11}{12}$

If A takes x days to build up the remaining structure, then $\frac{x}{8} = \frac{11}{12}$

$\therefore$ x = 22/3 days.

PIPES AND CISTERN

A pipe is connected to a tank or cistern. It is used to fill or empty the tank; accordingly, it is called an inlet or an outlet.

- Inlet pipe: A pipe which is connected to fill a tank is known as an inlet pipe.
- Outlet pipe: A pipe which is connected to empty a tank is known as an outlet pipe.

Important Formulae

1. If an inlet connected to a tank fills it in x hours, part of the tank filled in one hour is = $\frac{1}{s}$

2. If an outlet connected to a tank empties it in y hours, part of the tank emptied in one hour is = $\frac{1}{y}$

3. An inlet can fill a tank in x hours and an outlet can empty the same tank in y hours. If both the pipes are opened at the same time andy > x, the net part of the tank filled in one hour is given by;

$$= \left(\frac{1}{s} - \frac{1}{y}\right)$$

4. An inlet can fill a tank in x hours and another inlet can fill the same tank in y hours. If both the inlets are opened at the same time, the net part of the tank filled in one hour is given by;

$$= \left(\frac{1}{s} + \frac{1}{y}\right)$$

Solved examples:

1. Two pipes A and B can fill a tank in 12 and 24 minutes respectively. If both the pipes are used together, then how long will it take to fill the tank?

Solution –

Part filled by pipe A in 1 minute = $\frac{1}{12}$

Part filled by pipe B in 1 minute = $\frac{1}{24}$

Part filled by pipe A and pipe B in 1 minute

$$= \frac{1}{12} + \frac{1}{24} = \frac{1}{8}$$

$\therefore$ Both the pipe together can fill the tank in 8 minutes.

2. Pipes A and B can fill a tank in 5 and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in:

Solution –

Pipes A and B can fill the tank in 5 and 6 hours respectively. Therefore,
part filled by pipe A in 1 hour = $\frac{1}{5}$
part filled by pipe B in 1 hour = $\frac{1}{6}$

Pipe C can empty the tank in 12 hours. Therefore,
part emptied by pipe C in 1 hour = $\frac{1}{12}$

Net part filled by Pipes A,B,C together in 1 hour,

$$\frac{1}{5}+\frac{1}{6}-\frac{1}{12}=\frac{17}{60}$$

This is a positive number. This means rate of filling is greater than rate of emptying and so the tank can be filled in some hours.

∴ The tank can be filled in $\frac{60}{17} = 3\frac{9}{17}$ hours.

3. Two pipes A and B can fill a cistern in $37\frac{1}{2}$ minutes and 45 minutes respectively. Both pipes are opened. The cistern will be filled in just half an hour, if pipe B is turned off after what time?

Solution –

Pipe A alone can fill the cistern in $37\frac{1}{2} = \frac{75}{2}$ minutes.

B was closed after some minutes but A was open for 30 minutes.

Since A was open for 30 minutes, part of the cistern filled by pipe A = $\frac{2}{75} \times 30 = \frac{4}{5}$

So the remaining $= 1 - \frac{4}{5} = \frac{1}{5}$ part is filled by pipe B.

Pipe B can fill the cistern in 45 minutes. So, time required to fill $\frac{1}{5}$ part

$$= \frac{45}{5} = 9 \text{ minutes.}$$

∴ Pipe B is turned off after 9 minutes.

4. A water tank is two-fifth full. Pipe A can fill a tank in 12 minutes and pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely?

Solution –

Since pipe B is faster than pipe A, the tank will be emptied.

Part filled by pipe A in 1 minute = $\frac{1}{12}$

Part emptied by pipe B in 1 minute = $\frac{1}{6}$

Net part emptied by pipe A and B in 1 minute = $\frac{1}{6} - \frac{1}{12} = \frac{1}{12}$

So in 12 minutes, they will empty a full tank.

∴ Time taken to empty $\frac{2}{5}$ of the tank = $\frac{2}{5} \times 12$ = 4.8 min.

5. Three pipes A, B and C can fill a tank in 6 hours. After working at it together for 2 hours, C is closed and A and B can fill the remaining part in 7 hours. The number of hours taken by C alone to fill the tank is:

Solution –

A, B, C together can fill a tank in 6 hours.

∴ Part filled by pipes A,B,C together in 1 hr = $\frac{1}{6}$

All these pipes are open for only 2 hours and then C is closed.

Part filled by pipes A,B,C together in these 2 hours = $\frac{2}{6} = \frac{1}{3}$

Remaining part = $1 - \frac{1}{3} = \frac{2}{3}$

This remaining part of $\frac{2}{3}$ is filled by pipes A and B in 7 hours.

Therefore, part filled by pipes A and B in 1 hr = $\frac{\left(\frac{2}{3}\right)}{7} = \frac{2}{21}$

Part filled by pipe C in 1 hr

$= \frac{1}{6} - \frac{2}{21} \quad = \frac{3}{42} = \frac{1}{14}$

∵ C alone can fill the tank in 14 hours.

TIME , DISTANCE AND SPEED

Relationship between Speed, Distance and Time –

1. Speed $= \frac{\text{Dictance}}{\text{TiNe}}$
2. Distance = Speed × Time
3. Time $= \frac{\text{Dictance}}{\text{Speed}}$

When using these equations, it is important to keep the units straight. For instance, if the rate of the problem is given in kilometres per hour (kmph), then the time needs to be in hours , and the distance in kilometres. If the time is given in minutes, you will need to divide by 60 to convert it to hours before you can use the equation to find the distance in kilometres.

- **Convert kilometres per hour (km/hr) to metres per second (m/s)**

$$x \text{ km / hr} = (x \times \tfrac{1000}{3600}) \text{ m / s} = (x \times \tfrac{5}{18}) \text{ m / s}$$

- **Convert metres per second (m/s) to kilometres per hour (km/hr)**

$$x \text{ m / s} = (x \times \tfrac{3600}{1000}) \text{ km / hr} = (x \times \tfrac{18}{5}) \text{ km / hr}$$

❖ **Average Speed**

If an object covers a certain distance at x kmph and an equal distance at y kmph, the average speed of the whole journey $= \frac{2\,s\,y}{s+y}$ km / hr

❖ If the ratio of the speeds of A and B is a : b, then, the ratio of the time taken by them to cover the same distance is

$\frac{1}{a} : \frac{1}{b} = b : a$

❖ Relative Speed

- If two objects are moving in the same direction at v_1 m/s and v_2 m/s respectively where $v_1 > v_2$, then their relative speed = ($v_1 - v_2$) m / s
- If two objects are moving in opposite directions at v_1 m/s and v_2 m/s respectively, then their relative speed = ($v_1 + v_2$) m/s

Solved examples:

1. a) Express speed of 72 km / hr in m / s .
b) Express speed of 25 m / s in km / hr .

Solution –

a) 72 km / hr = $(72 \times \frac{5}{18})$ m / s = 20 m / s

b) 25 m / s = $(25 \times \frac{18}{5})$ km / hr = 90 km / hr

2. A person crosses a 600 metre long street in 5 minutes. What is his speed in km per hour?

Solution –

Distance =600 metre = 0.6 km
Time = 5 minutes = $\frac{1}{12}$ hour

Speed $= \frac{\text{Dictance}}{\text{TiNe}} = \frac{0.6}{1/12}$ = 7.2 km/hr.

3. A man completes a journey in 10 hours. He travels first half of the journey at the rate of 21km/hr and second half at the rate of 24 km/hr. Find the total journey in km.

Solution –

Average Speed = $\frac{2sy}{s+y}$
$= \frac{2 \times 21 \times 24}{21 + 24}$ km / hr = 22.4 km / hr

Total distance = 22.4×10=224 km.

4. In covering a distance of 30 km, Arun takes 2 hours more than Anil. If Arun doubles his speed, then he would take 1 hour less than Anil. What is Arun's speed?

Solution –

Let speed of Arun = x kmph,
speed of Anil = y kmph
distance = 30 km

We know that Time $= \frac{\text{Dictance}}{\text{Speed}}$ Hence,

$$\frac{30}{s} - \frac{30}{y} = 2 \cdots (1)$$

$$\frac{30}{y} - \frac{30}{2s} = 1 \cdots (2)$$

Adding (1)and (2)

$$\frac{30}{s} - \frac{30}{2s} = 3$$
$$\Rightarrow \frac{30}{2s} = 3$$
$$\Rightarrow \frac{15}{s} = 3$$
$$\Rightarrow \frac{5}{s} = 1$$
$$\Rightarrow x = 5$$

$\therefore$ Arun's speed = 5 kmph.

5. A man travelled a distance of 61 km in 9 hours. He travelled partly on foot at 4 km/hr and partly on bicycle at 9 km/hr. What is the distance travelled on foot?

Solution-

Let the time in which he travelled on foot = x hr
Then the time in which he travelled on bicycle = (9−x) hr

Distance = speed × time

$$\Rightarrow 4x + 9(9-x) = 61$$
$$\Rightarrow 4x + 81 - 9x = 61$$
$$\Rightarrow 5x = 20$$
$$\Rightarrow x = 4$$

$\therefore$ Distance travelled on foot = 4x = 16 km.

Unit – 4: Concept of Angles, Different Polygons like triangles, rectangle, square, right angled triangle, Pythagorean Theorem, Perimeter and Area of Triangles, Rectangles, Circles

Concept of Angle

In plane geometry, an **angle** is the figure formed by two rays, called the *sides* of the angle, sharing common endpoint, called the vertex of the angle.

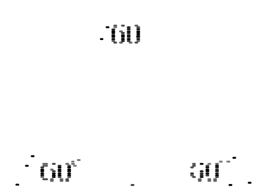
- Angles smaller than 90°are called **acute angles.**
- An angle equal to 90° is called a **right angle.**
- Angles larger than a right angle and smaller than 180° are called **obtuse angles**.

Triangle

A **triangle** is a polygon with three edges and three vertices. The sum of the 3 angles of a triangle is 180°.

Types of triangle:

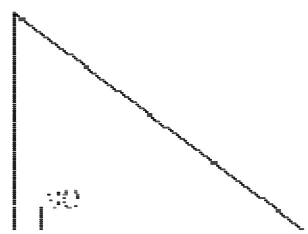
1. An **equilateral triangle** has all sides the same length. An equilateral triangle is also a regular polygon with all angles measuring 60°.



2. An **isosceles triangle** has two sides of equal length. An isosceles triangle also has two angles of the same measure, namely the angles opposite to the two sides of the same length.

3. A **scalene triangle** has all its sides of different lengths. Equivalently, it has all angles of different measure.

4. A **right triangle** (or right-angled triangle) has one of its interior angles measuring 90° (a right angle). The side opposite to the right angle is the hypotenuse, the longest side of the triangle.



Perimeter and area

Let the length of the three sides of a triangle be a , b , c. Then,

- Perimeter of the triangle = a + b + c
- Area of the triangle $= \frac{1}{2} \times \text{base} \times \text{height}$
- Area = $\sqrt{s(s-a)(s-b)(s-c)}$ (if all the sides are given)

  Where $s = \frac{1}{2}(a + b + c)$

❖ Let the side of an equilateral triangle be a . Then ,

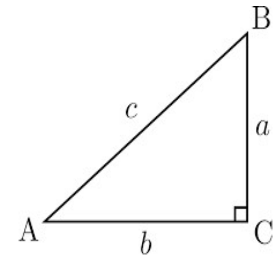
➢ Perimeter of equilateral the triangle = 3 a

➢ Area of the equilateral triangle $= \frac{\sqrt{3}}{4} \times a^2$

Pythagorean Theorem

It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

$$\boxed{a^2 + b^2 = c^2}$$



Solved examples:

1. Find the perimeter of the triangle having length of the sides 5 cm, 8 cm, 7 cm.

Solution:

Let a , b, c be the length of the sides of the given triangle.
So a = 5 cm, b = 8 cm, c = 7 cm.
Perimeter of the triangle = (a + b + c) cm
= (5 + 8 + 7) cm
= 20 cm.

∴ Perimeter of the given triangle is 20 cm.

2. Find the area of the triangle whose sides measure 13 cm , 14 cm , 15 cm.

Solution –

Let a = 13 cm , b = 14 cm , c = 15 cm.

Then $s = \frac{1}{2}(a + b + c)$

$= \frac{1}{2}(13 + 14 + 15) = 21$ cm.

Area of the triangle = $\sqrt{s\,(s - a)(s - b)(s - c)} = \sqrt{21\,(21 - 13)(21 - 14)(21 - 15)}$

$= \sqrt{21 \times 8 \times 7 \times 6}$

= 84 cm^2

∴ Area of the triangle is 84 cm^2.

3. Let the perimeter of an equilateral triangle be 27 cm. Then find the length of its sides.

Solution:

Given perimeter of the equilateral triangle = 27 cm

Let a be the length of the side of the equilateral triangle.

$\Rightarrow 3a = 27$

$\Rightarrow a = \frac{27}{3} = 9$ cm

$\therefore$ Length of the side of the equilateral triangle is 9 cm.

4. Find the area of the equilateral triangle having sides of length 8 cm.

Solution:

Given length of the side of the equilateral triangle = 8 cm

Area of the triangle $= \frac{\sqrt{3}}{4} \times a^2$

$$= \frac{\sqrt{3}}{4} \times (8)^2$$

$$= 16\sqrt{3} \text{ cm}^2$$

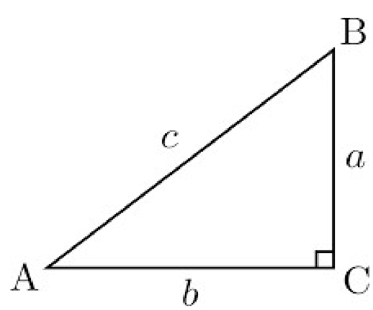
$\therefore$ Area of the given equilateral triangle is $16\sqrt{3}$ cm^2.

5. Find the length of the hypotenuse of the right angled triangle when the length of the base and the perpendicular are 5 cm and 12 cm respectively.

Solution:

Given length of the base (b) = 5 cm

length of the perpendicular (a) = 12 cm



Using Pythagorean theorem, $\mathbf{a^2 + b^2 = c^2}$

$\Rightarrow c^2 = 5^2 + 12^2$

$\Rightarrow c^2 = 25 + 144$

$\Rightarrow c^2 = 169$

$\Rightarrow c = 13$ cm

$\therefore$ Length of hypotenuse of the triangle is 13 cm.

6. The base of a triangular field is three times its altitude. If the cost of cultivating the field at Rs. 24.68 per hectare be Rs.333.18, find its base and height.

Solution –

Area of the field $= \frac{\text{total cost}}{\text{rate}} = \frac{333.18}{24.68} = 13.5$ hectares [1 hectare = 100m × 100m]

$= (13.5 \times 10000)\ \text{m}^2 = 135000\ \text{m}^2$

Let altitude = x metres and base = 3x metres.

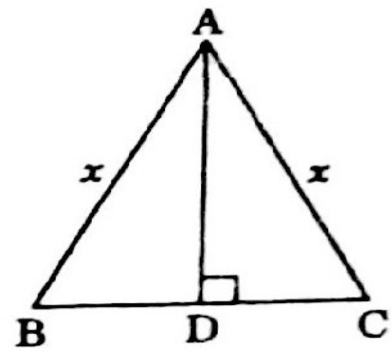
Then, $\frac{1}{2} \times 3x \times x = 135000$

$\Rightarrow x^2 = 90000$

$\Rightarrow x = 300$ metres

Thus, base = 900 metres and altitude = 300 metres.

7. The altitude drawn to the base of the isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.



Solution:

Let ABC be a isosceles triangle and AD be its altitude.
Let AB = AC = x cm. Then BC = (32 – (AB + AC))
= (32 – 2x) cm

Since in an isosceles triangle the altitude bisects the base, so we have BD = CD = (16 – x) cm

In triangle ADC , $AD^2 + BD^2 = AB^2$

$\Rightarrow x^2 = 8^2 + (16 - x)^2$

$\Rightarrow x^2 = 64 + 256 + x^2 - 32x$

$\Rightarrow 32x = 320$

$\Rightarrow x = 10$

So BC = (32 – 2x) =(32 – 20) = 12 cm

Now Area of the triangle = $\frac{1}{2} \times \text{base} \times height$

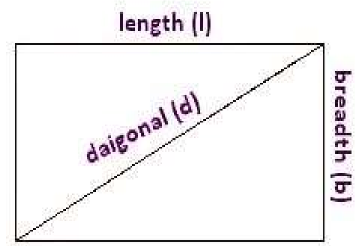
$= \frac{1}{2} \times 12 \times 8$

= 48 cm^2

$\therefore$ Area of the triangle is 48 cm^2.

<u>Rectangle</u>

A **rectangle** is a quadrilateral with four right angles. In a rectangle both pairs of opposite sides are parallel and equal in length. The sum of the angles of a rectangle is 360°.



Perimeter and area

If a rectangle has length l and width b,

- it has perimeter $P = 2\,(l + b)$
- it has area $A = l \times b$

Solved examples:

1. Find the area and the perimeter of a rectangle having length 12 cm and breadth 10cm.

Solution –

Given Length of the rectangle = 12 cm

Breadth of the rectangle = 10 cm

Perimeter $= 2\,(l + b) = 2\,(12 + 10)$

$= 44$ cm.

Area $= l \times b = 12 \times 10 = 120\ \mathrm{cN}^{2}$.

∴ Area and perimeter of the triangle are 120 cm^2 & 44 cm respectively.

2. A field is in the form of a rectangle having its sides in the ratio 2 : 3. The area of the field is 1.5 hectares. Find the length and breadth of the field.

Solution:

Let length = 2x and breadth = 3x metres.

Area = 1.5 hectares $= 1.5 \times 10000\ \mathrm{m}^2 = 15000\ \mathrm{m}^2$. [1 hectare = 100m × 100m]

$\Rightarrow 2x \times 3x = 15000$

$\Rightarrow 6x^2 = 15000$

$\Rightarrow x^2 = 2500$

$\therefore$ x $= 50$ metres

Length = 2x = 100 metres and breadth = 3x = 150 metres.

∴ The length and breadth of the rectangle is 100 m & 150 cm respectively.

3. If three angles of a quadrilateral are 50°, 75° and 80°, then find the remaining angle of the quadrilateral.

Solution:

Given three angle of the quadrilateral are 50°, 75° and 80°.

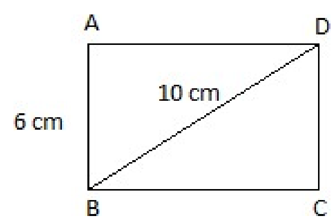
We know that the sum of the angles of a quadrilateral is 360°.

Hence the remaining angle = $360° - (50° + 75° + 80°)$

$= 360° - 205°$

$= 155°$

4. Let one side and a diagonal of a rectangle be 6 cm & 10 cm respectively, then find the area of the rectangle.



Solution:

Given breadth of the rectangle = 6 cm

Length of the diagonal = 10 cm

Δ ABD is a right angle triangle. So we have $BD^2 = AD^2 + AB^2$

$\Rightarrow 10^2 = AD^2 + 6^2$

$\Rightarrow 100 = AD^2 + 36$

$\Rightarrow AD^2 = 64$

$\Rightarrow AD = 8$ cm

Area of ABCD = AD × AB

$= 6 \times 8 = 48\ cN^2$

∴ Area of the given rectangle is 48 cm^2.

5. The length of a rectangle is 8 cm and the width is 5 cm. If the length is greater by 2 cm, what should the width be so that the new rectangle has the same area as the first one?

Solution:

Given length of the rectangle = 8 cm

Breadth of the rectangle = 5 cm

So area of the rectangle = $l \times b = 8 \times 5 = 40 cm^2$

If the length is increased by 2 cm, then the new length of the rectangle = 10 cm.

Area = $l \times b$

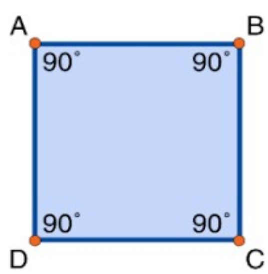
$\Rightarrow 40 = 10 \times b$

$\Rightarrow b = 4$ cm.

$\therefore$ The new width of the rectangle is 4 cm.

Square

A **square** is a regular quadrilateral, which means that it has four equal sides and four equal angles (i.e.90°).



Perimeter and area

- The perimeter of a square whose four sides have length l is P = 4 l
- And the area $A = l^2$

Solved examples:

1. Find the area and the perimeter of the square having sides 6 cm.

Solution:

Given length of the side $l = 6$ cm

So perimeter $P = 4\,l$

$= 4 \times 6 = 24$ cm

Area $A = l^2$

$\Rightarrow A = 6^2 = 36\ \text{cm}^2$

$\therefore$ Perimeter and area of the given square is 24 cm & 36 cm^2 respectively.

2. Find the perimeter of the square whose area is 100 cm^2.

Solution:

Let the sides of the square be a cm.

Given area of the square = 100 cm^2.

$\Rightarrow a^2 = 100$

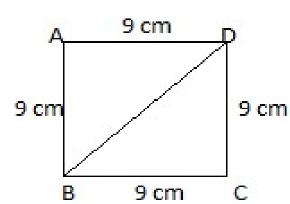
$\Rightarrow a = 10$ cm.

Perimeter $= 4 \times a = 4 \times 10 = 40$ cm

$\therefore$ Perimeter of the square is 40 cm.

3. Find the length of the diagonal of the square having sides 9 cm.

Solution:



Let ABCD be a square having sides 9 cm.

Δ ABD is a right angle triangle. So we have $BD^2 = AD^2 + AB^2$

$\Rightarrow BD^2 = 9^2 + 9^2$

$\Rightarrow BD^2 = 162$

$\Rightarrow BD = 9\sqrt{2}$ cm

$\therefore$ Length of the diagonal is $9\sqrt{2}$ cm.

- If a is the length of the side of a square, then its diagonal = $a\sqrt{2}$
- If d is the length of the diagonal of a square ,then the length of its sides = $\frac{d}{\sqrt{2}}$

4. Find the area of a square whose diagonal is 4 cm.

Solution:

Given length of the diagonal of a square = 4 cm

Length of its sides = $\frac{d}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ cm

Area of the square = $2\sqrt{2} \times 2\sqrt{2} = 8$ cm²

∴ Area of the square is 8 cm^2.

- ❖ Area of square = $\frac{1}{2}(d)^2$ (if length of the diagonal is given)

5. The perimeter of a square courtyard is 100 m. find the cost of cementing it at the rate of Rs. 5 per m^2.

Solution:

Perimeter of square courtyard = 100 m

Therefore, side of the square courtyard = $\frac{100}{4}$ = 25 m

Therefore, area of square courtyard = (25 × 25) m^2 = 625 m^2

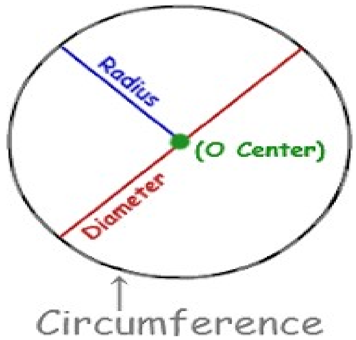
For 1 m^2, the cost of cementing = Rs. 5

For 625 m^2, the cost of cementing = 625 × 5 = Rs. 3125

Circle

A **circle** is a simple closed shape. It is the set of all points in a plane that are at a given distance from a given point, the centre; equivalently it is the curve traced out by a point that moves so that its distance from centre is constant.

- A circle has a total of **360 degrees** all the way around the center,



Perimeter and area

Let the radius of a circle be r. Then,

- Perimeter/Circumference of the circle = 2πr
- Area of the circle = π r ^{2}

Solved examples:

1. Find the circumference of a circle having radius 21cm.

Solution:

Given r = 21 cm

Circumference = 2πr

$$= 2 \times \frac{22}{7} \times 21$$

= 132 cm.

∴ Circumference of the circle is 132 cm.

2. Find the area of the circle having radius 7 cm.

Solution:

Given radius of the circle (r) = 7 cm

Area = π r ^{2}

$= \frac{22}{7} \times 7^2 = 154\ cm^2$.

∴ Area of the circle is 154 cm^2.

3. A wheel makes 200 revolutions in covering a distance of 44 km. Find the radius of the wheel.

Solution:

Distance covered in 200 revolutions = 44 km = (44×1000) N = 44000 m

$\Rightarrow$ Distance covered in 1 revolution $= \frac{44000}{200} = 220$ N

Distance covered in 1 revolution = perimeter of the wheel

$\Rightarrow$ $2nr = 220$

$\Rightarrow$ $2 \times \frac{22}{7} \times r = 220$

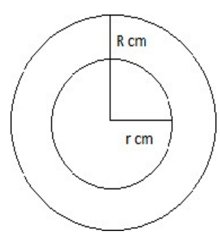
$\Rightarrow$ $r = 220 \times \frac{7}{22} \times \frac{1}{2}$

$\Rightarrow$ $r = 35$ cN.

∴ Radius of the wheel is 35 cm.

4. Two concentric circles form a ring. The inner and the outer circumference of the ring are 132 cm & 176 cm respectively. Then find the width of the ring.

Solution:



Let R and r be the radii of the outer and inner circles respectively.
According to the question, for the outer circle, $2nR = 176$ cN

$\Rightarrow$ $2 \times \frac{22}{7} \times R = 176$

$\Rightarrow$ $R = 176 \times \frac{7}{22} \times \frac{1}{2}$

$\Rightarrow$ R = 28 cm

Similarly for the inner circle, $2nr = 132$

$\Rightarrow$ $2 \times \frac{22}{7} \times r = 132$

$\Rightarrow$ $r = 132 \times \frac{7}{22} \times \frac{1}{2}$

$\Rightarrow$ $r = 21$ cN

∴ Width of the ring = R – r = 28 cm – 21 cm = 7 cm.

5. If the radius of a circle is decreased by 50%, then find the % decrease in area.

Solution:

Let the radius of a circle = r cm

So area of the circle = $n\,r^2$

Now if the radius decreases by 50% the new radius (R) = 50% of r

$$= \frac{50}{100} \times r = \frac{r}{2}$$

The new area = $n\,R^2 = n\left(\frac{r}{2}\right)^2 = \frac{n\,r^2}{4}$

Decrease in area = $\frac{\text{change in area}}{\text{originaS area}} \times 100 = \frac{n\,r^2 - \frac{G\,r^2}{4}}{n\,r^2} \times 100 = \frac{\frac{3G\,r^2}{4}}{n\,r^2} \times 100 = 75\%$

∴ Decrease in area = 75%.

Unit – 5: Raw and Grouped Data, Bar Graphs, Pie charts, Mean, Median and Mode, Events and Sample Space, Probability

Data analysis is an important aspect of almost every competitive exam today. Usually, a table or a bar diagram or a pie chart or a sub-divided bar diagram or a graph is given and candidates are asked questions that test their ability to analyze the data given in those forms.

Raw data

Raw data or primary data are collected directly related to their object of study (statistical units). When people are the subject of an investigation, we may choose the form of a survey, an observation or an experiment.

Examples:

Let us consider the marks secured by 20 students of a class (total mark 500)

453	301	220	485	211
420	143	388	357	229
98	448	429	190	150

490	324	256	373	389

This is called raw data i.e. not processed; only gathered.

Grouped data

Grouped data are data formed by aggregating individual observations of a variable into groups, so that a frequency distribution of these groups serves as a convenient means of summarizing or analyzing the data.

Example:

One way of arranging the above data is as below

Marks	Number of students
0 – 100	1
100- 200	3
200 - 300	4
300 – 400	6
400 - 500	6

This is called grouping of data i.e. arranging data in a particular way.

Bar Graphs

A **bar graph** (also called bar chart) is a graphical display of data using bars of different heights.

We can use bar graphs to show the relative sizes of many things, such as what type of car people have, how many customers a shop has on different days and so on.

Solved examples:

1. Study the following graph and answer the questions.

QA. From the graph calculate the sum of crude oil imports in the years 1991 & 1996.

Solution: Crude oil import in the year 1991 = 35 lakh barrels

Crude oil import in the year 1996 = 45 lakh barrels

Sum = 35 + 45 = 80 lakh barrels.

QB. Calculate the average crude oil imports by India during 1990 – 1996.

Solution: sum of crude oil imports during 1990 – 1996 = 10 + 35 + 30 + 40 + 30 + 25 + 45

= 215 lakh barrels.

Hence, average = $\frac{215}{7}$ = 30.71 lakh barrels.

QC. In which year(s) the crude oil import was greater than average import?

Solution:

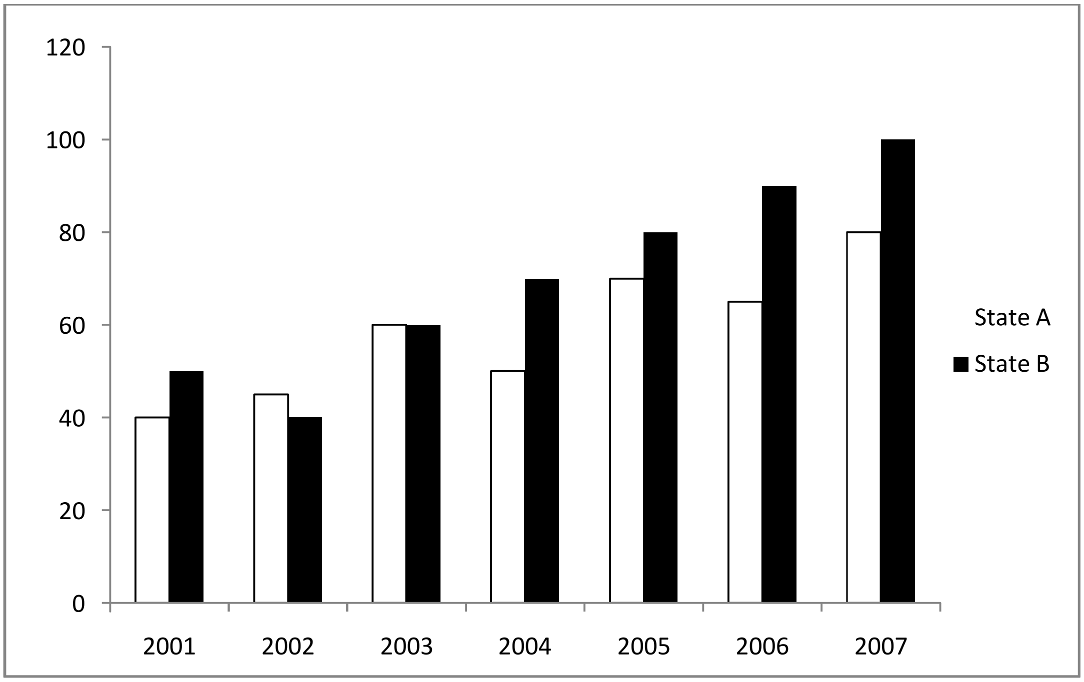
From above we have average = 30.71 lakh barrels

Hence, in the years 1991, 1993, 1996 the crude oil import was greater than the average.

2. The following bar graph shows the population of two states A & B in lakhs.

Answer the following questions based on the data given below.

Population of two states (in lakhs) over the years



Number of years

Q A: Approximately, what is the average population of state A for all the given years?

Solution:

Total population of State A = 410 Lakhs

Average Population of State A = $\frac{\text{Total population of State A}}{\text{Number of Years}} = \frac{40+45+60+50+70+65+80}{7}$

$= \frac{410}{7} = 58.57$ Lakhs

Therefore, The approx average population of State A is 59 Lakhs.

Q B: What is the ratio of the total population of state A for the years 2001, 2002 and 2003 together to the population of state B for 2005, 2006 and 2007 together?

Solution:

As we need to find ratio, State A (2001 + 2002 + 2003) : State B(2005 + 2006 +2007)

= (40 + 45 + 60) : (80+ 90+100)

= 145: 270 = 29:54

Therefore the ratio of number of people in the state A (2001 + 2002 + 2003) to the number of people in the state B (2005 + 2006+ 2007) is 29 : 54.

Q C: What is the percentage rise in population of state B from the year 2003 to 2004?

Solution:

As we need to find the percentage rise in the population,

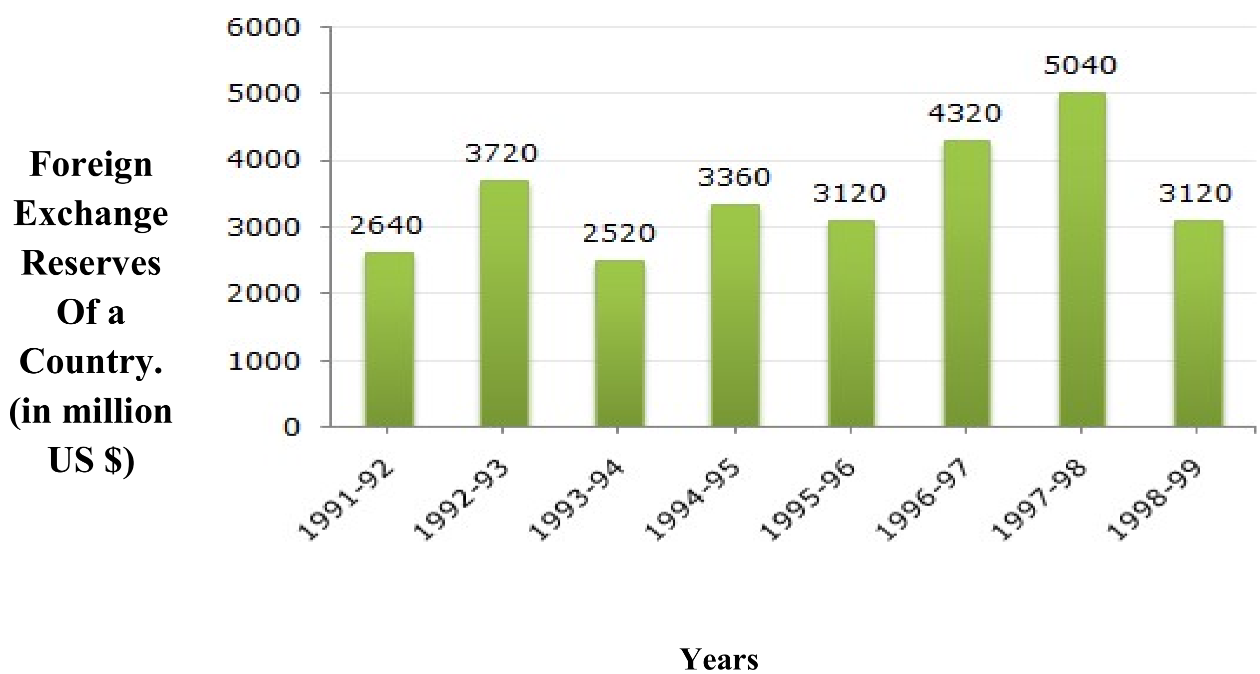
Percentage change = $\frac{\text{change in value}}{\text{reference value}} \times 100$

So, Percentage Increase $= \frac{70-60}{60} \times 100$

$$= \frac{10}{60} \times 100 = 16.66\%$$

Therefore, there has been a rise in 16.66% of population during the year 2003 and 2004 in State B.

3. The bar graph given below shows the foreign exchange reserves of a country (in million US $) from 1991 - 1992 to 1998 - 1999.



Q A. Calculate the average foreign reserves of the country from 1991 - 1992 to 1998 – 1999.

Solution:

Sum of foreign reserves of the country = (2640 + 3720 + 2520 + 3360 + 3120 + 4320 + 5040 + 3120) million US $.

`   = 27840 million US $.

Hence, average = $\frac{\text{cuN}}{8} = \frac{27840}{8} = 3480$ million US \$.

Q **B.** The foreign exchange reserves in 1997-98 were how many times that in 1994-95?

Solution:

Required ratio = $= \frac{\text{foreign exchange reserves in 1997−98}}{\text{foreign exchange reserves in 1994−95}} = \frac{5040}{3360} = 1.5$.

Q C. What was the percentage increase in the foreign exchange reserves in 1997-98 over 1993-94?

Solution:

Foreign exchange reserves in 1997 - 1998 = 5040 million US \$.

Foreign exchange reserves in 1993 - 1994 = 2520 million US \$.
Increase = (5040 – 2520) = 2520 million US \$.

Percentage increase = $(\frac{2520}{2520} \times 100)\ \% = 100\%$.

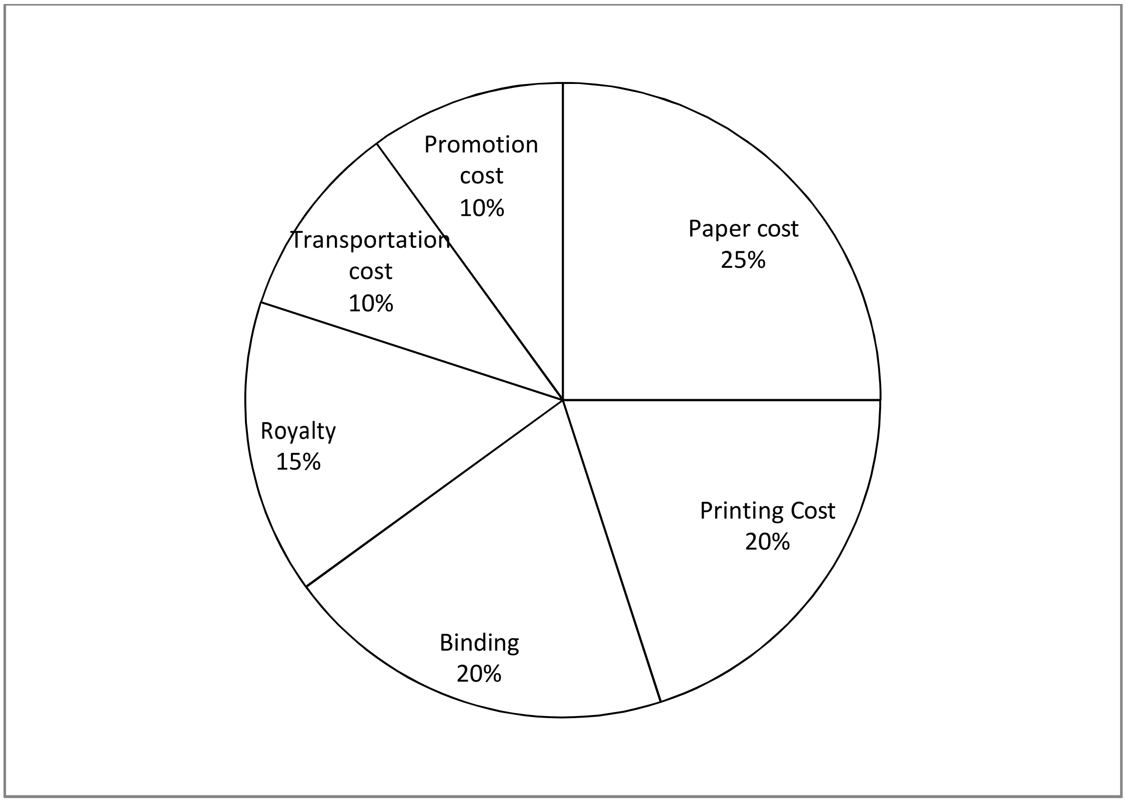
Pie chart

A pie is a baked dish which is round in shape. A pie chart or pie graph is a special chart that uses "pie slices" to show relative sizes of data. The chart is divided into sectors, where each sector shows the relative size of each value.

Solved examples:

1. The following pie-chart shows the percentage distribution of the expenditure incurred in publishing a book. Study the pie-chart and the answer the questions based on it.

Various Expenditures (in percentage) Incurred in Publishing a Book



QA. If for a certain quantity of books, the publisher has to pay Rs. 30,600 as printing cost then, what will be amount of royalty to be paid for these books?

Solution:

Given that the printing cost = Rs. 30,600

Let the royalty to be paid be Rs. x.

So we can write, $\frac{20\% \text{ of totaS coct}}{15\% \text{ of totaS coct}} = \frac{30,600}{s}$

$\Rightarrow 20 \times x = 15 \times 30,600$

$\Rightarrow x = \frac{15 \times 30,600}{20} = \text{Rs. } 22,950$

Hence royalty to be paid = Rs. 22,950

Q B. Promotion cost on the book is less than the paper cost by what percentage?

Solution:

Promotion cost of book = 10% of C.P.

paper cost on book = 25% of C.P.

Difference = (25% of C.P.) - (10% of C.P) = 15% of C.P.

Percentage difference = $\left(\frac{\text{Difference}}{\text{paper coct}} \times 100\right)\%$

$= \left(\frac{15\ \%\ \text{of C.P}}{25\%\ \text{of €.P.}} \times 100\right)\% = 60\%$.

QC . If the total cost of printing a certain quantity of books is Rs.2,50,000. Then find the sum of its transportation cost, promotion cost and binding cost.

Solution:

Given that the total cost = Rs.2,50,000

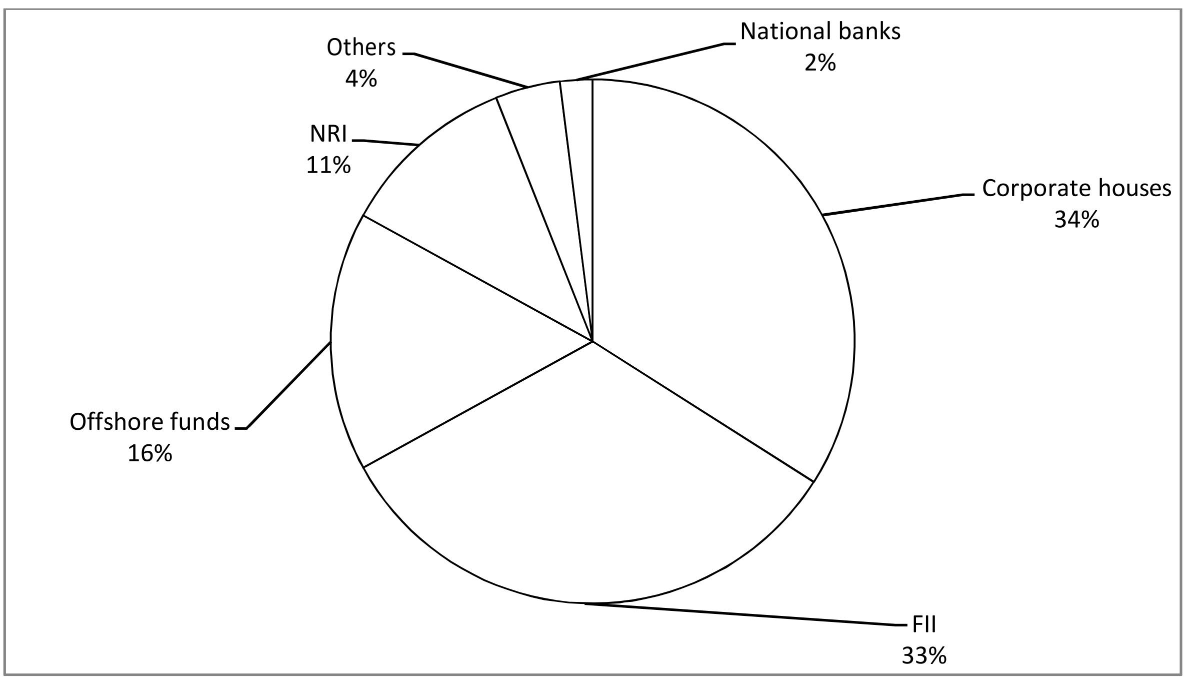
Sum = (10 % of total cost) + (10 % of total cost) + (20 % of total cost)

= (10% + 10% + 20%) of 2,50,000

$= \frac{40}{100} \times 2{,}50{,}000$

= Rs. 1,00,000

2. The following pie chart shows the amount of subscriptions generated for India bonds for different categories of investors.



Q A. If the investments by NRIs are Rs 1100 crore, then the investment by corporate houses and FIIs together is:

Solution:

Let the investment by corporate houses and FIIs together is be x.

So we can write, $\frac{11\% \text{ of totaS investments}}{34\% \text{ of totaS investments} + 33\% \text{ of totaS investments}} = \frac{1100}{s}$

⇒ $\frac{11}{34+33} = \frac{1100}{s}$

⇒ $x = \frac{1100 \times 67}{11} = 6700$ crore

The investment by corporate houses & FII is = 6700 crore.

Q B. What is the approximate ratio of investment flows into India Bonds from NRIs to corporate houses?

Solution:

Investment flows into India Bonds from NRIs = 11%

Investment flows into India Bonds from corporate houses = 34%

Required ratio = 11 : 34 = 1: 3(approx.)

QC. In the corporate sector, how many degrees should be there in the central angle?

Solution:

From the above pie chart we have corporate sector = 34% of total subscriptions

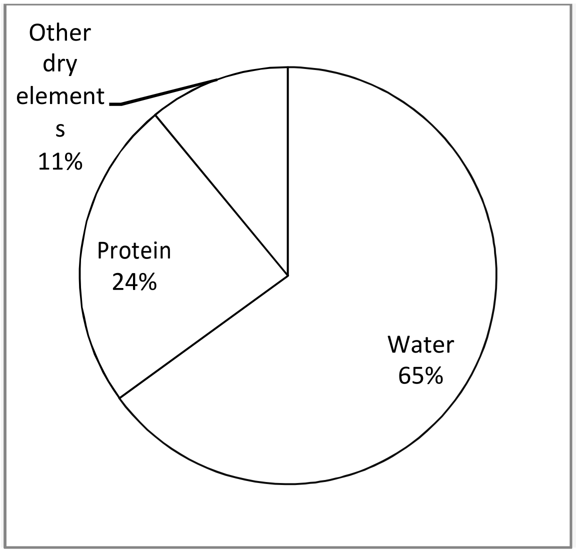
As we know a circle has a total of 360 degrees.

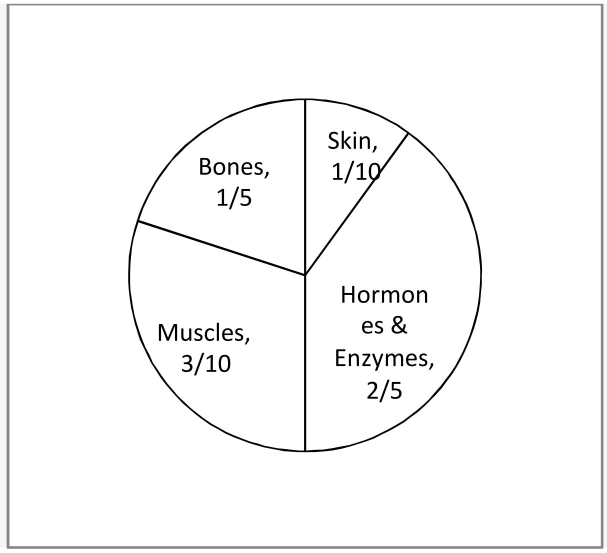
So, 100 % of total subscriptions = 360°

⇒ 1 % of total subscriptions = 3.6°

⇒ 34% of total subscriptions = $34 \times 3.6 = 122.4°$.

3. The following pie charts Figures (a) and (b) give the information about the distribution of weight in the human body according to different kinds of components. Study the pie charts carefully and answer the question given.





QA. What percentage of proteins of the human body is equivalent to the weight of its skin?

Solution:

Proteins contribute 24% of human weight.

Skin contributes $= \frac{1}{10} \times 100 = 10\%$ of human weight.

Let's say weight of human body is 100 kg.

$\therefore$ Weight of proteins = 24% of 100 kg = 24 kg and

Weight of skin = 10% of 100 kg = 10kg

Now 10 kg is equivalent to $\frac{10}{24} \times 100\% = 41.66\%$.

So 41.66% of proteins of the human body is equivalent to the weight of its skin.

Q B. If the skin weighs 12 kg, then find the weight of water.

Solution:

Let the weight of water be x kg.

Given that skin weighs 12 kg

$$\therefore \frac{\frac{1}{10}\text{ of totaS body weight}}{\frac{65}{100}\text{ of totaS body weight}} = \frac{12}{s}$$

$$\therefore x = 12 \times \frac{65}{100} \times 10 = 78 \text{ kg.}$$

Now weight of water = 78 kg.

QC. If the total weight of the body is 47 kg, then find out the difference between weight of bones and other dry elements.

Solution:

Given total weight of the body is 47 kg

Weight of bones = $\frac{1}{5} \times 47 = 9.4$ kg

Weight of dry elements = $\frac{11}{100} \times 47 = 5.17$ kg.

Difference = 9.4 kg – 5.17 kg = 4.23 kg

Mean, Median & Mode

Mean:

Mean is basically the average found by adding all data values and dividing by the number of data values.

The mean (m) of a sample of n values $x_1, x_2, x_3, x_4, \ldots\ldots\ldots x_n$, is

$$\boxed{m = \frac{x1 + x2 + x3 + x4 + \ldots + xn}{n}}$$

Examples:

1. Find the mean of 5, 11,16,10,18.

Solution:

Mean (m) = $\frac{x1 + x2 + x3 + x4 + \ldots + xn}{n} = \frac{5+11+16+10+18}{5} = \frac{60}{5} = 12$

2. Find the mean of 1.2, 3.15, 4.19, 4.22, 5.75, 1.90.

Solution:

m = $\frac{x1 + x2 + x3 + x4 + \ldots + xn}{n} = \frac{1.2+3.15+4.19+4.22+5.75+1.90}{6} = 3.4$

3. Find the mean of first five multiples of 8.

Solution:

The first five multiples of 8 are 8, 16, 24, 32, and 40.

Mean = $\frac{8+16+24+32+40}{5} = \frac{120}{5} = 24$.

4. There are two sections A and B of a class, consisting of 25 and 32 students' respectively. If the average weight of section A is 45kg and that of section B is 43kg, find the average weight of the whole class.

Solution:

Average weight of 25 students of section A = 45 kg

∴ Total weight of 25 students of section A = 45 $\times 25 = 1125$ kg

Average weight of 32 students of section B = 43 kg

∴ Total weight of 32 students of section A = 43 $\times 32 = 1376$ kg

Now average weight of whole class = $\frac{\text{totaS weight}}{\text{totaS no of ctudentc}} = \frac{1125+1376}{25+32} = \frac{2501}{57} = 43.87$ kg.

5. The mean of 25 numbers is 48. If two numbers, 15 and 17 are discarded, then find the mean of the remaining numbers.

Solution:

Average of 25 numbers = 48

∴ Sum = 48 $\times 25 = 1200$

If 15 and 17 are discarded then new sum = 1200 – (15 + 17) =1168

New mean = $\frac{1168}{23} = 50.78$.

Median:

Median is the middle value for a set of data that has been arranged in order of smallest to largest.

To find median of **n** no. of values first we have to arrange the given values in ascending order.

Median = the value in $(\frac{n+1}{2})$th place (if n is odd)

$= \frac{\text{the value in } (\frac{n}{2})\text{th place} + \text{the value in } (\frac{n}{2}+1)\text{th place}}{2}$ (if n is even)

Solved examples:

1. Find the median for the following data.

96, 37, 14, 35, 55, 110, 24

Solution:

Arranging the above data we have,

14, 24, 35, 37, 55, 96,110

Here n= 7(odd)

So, median = the value in $(\frac{n+1}{2})$th place = the value in 4^{th} place = 37.

2. Find the median for the following data set:

140, 27, 38, 23, 69, 121, 15, 52.

Solution:

Arranging the above data we have,

15, 23, 27, 38, 52, 69, 121, 140

Here n= 8(even)

So median $= \frac{\text{the value in } (\frac{n}{2})\text{th place} + \text{the value in } (\frac{n}{2}+1)\text{th place}}{2}$

$= \frac{38+52}{2} = 45.$

3. Find the median for the following data set:
36, 53, 1.2, 3.9, 47,12.6

Solution:

Arranging the above data we have,

1.2, 3.9, 12.6, 36, 47, 53

Here n=6(even)

So, median $= \frac{\text{the value in } (\frac{n}{2})\text{th place} + \text{the value in } (\frac{n}{2}+1)\text{th place}}{2}$

$= \frac{12.6+36}{2} = 24.3.$

4. Find the median for a set containing squares of all the even numbers between 1 to 20.

Solution:

We need to find median of squares of the set (2, 4, 6, 8, 10, 12, 14, 16, 18)

Here n = 9 (odd)

So desired value is square of 5^{th} value = 10^2= 100

5. Find the median $\frac{3}{5}, \frac{1}{2}, \frac{12}{13}, \frac{6}{5}, \frac{1}{6}, \frac{7}{9}$.

Solution:

Now to arrange the above fractions in ascending order first we have to a common multiple of all the denominators.

So the common multiple of 5, 2, 13, 5, 6, 9 is 1170

$$\frac{3}{5} = \frac{3 \times 234}{5 \times 234} = \frac{702}{1170}$$

$$\frac{1}{2} = \frac{1 \times 585}{2 \times 585} = \frac{585}{1170}$$

$$\frac{12}{13} = \frac{12 \times 90}{13 \times 90} = \frac{1080}{1170}$$

$$\frac{6}{5} = \frac{6 \times 234}{5 \times 234} = \frac{1404}{1170}$$

$$\frac{1}{6} = \frac{1 \times 195}{6 \times 195} = \frac{195}{1170}$$

$$\frac{7}{9} = \frac{7 \times 130}{9 \times 130} = \frac{910}{1170}$$

Now arranging the above data according to the above values we have,

$\frac{1}{6}, \frac{1}{2}, \frac{3}{5}, \frac{7}{9}, \frac{12}{13}, \frac{6}{5}$

So median $= \dfrac{\text{the value in } (\frac{n}{2})\text{th place} + \text{the value in } (\frac{n}{2} + 1)\text{th place}}{2}$

$= \dfrac{\frac{3}{5} + \frac{7}{9}}{2} = \dfrac{31}{45}.$

M
o
d
e

Mode is the most frequent value in a data set. There can be no mode, one mode or multiple modes in a data set.

Solved examples:

1. Find the mode of the following set of scores.

    4, 2, 8, 9, 2, 7,2,11

Solution:

Mode = 2,since it occurs 3 times in the given data set, which is more than any other value.

2. Find the mode of the following data set.

   53, 23, 18, 23, 23, 96, 84, 53, 107, 88, 53.

Solution:

Mode = 53 and 23 (both occurs most frequently i.e. 3 times)

3. The following frequency table shows the marks obtained by students in a quiz. Given that 3 marks is the only mode, what is the least value for *x*?

Marks	1	2	3	4	5	6
Number of students	15	13	x	9	7	3

Solution:

Given that 3 is the only mode of the data set.

∴ *x* is at least 16 (if *x* is less than 16 then 3 will not be the mode)

4. Find the mode of the following data set.

   1020, 962, 321, 1020, 420, 786, 962, 420, 962, 333,469,656.

Solution:

1020, 962,420 are the modes. (Each occurs most frequently i.e. 2 times)

5. A marathon race was completed by 5 participants. What is the mode of times taken by them? (given in hours)

   2.7 , 8.3, 3.5, 5.1, 4.9

Solution:

Ordering the data in ascending order we get,

2.7, 3.5, 4.9, 5.1, 8.3

Since each value occurs only once in the data set, there is no mode for this set of data.

Probability

Probability is a measurement of uncertainty.

- Random experiment:

  A random experiment is an experiment or a process for which the outcome cannot be predicted with certainty and the process repeatedly occurs under homogeneous conditions.

- Outcome:

An **outcome** is a result of a random experiment.

- Sample space(S):

The set of all possible outcomes is called the **sample space**.

- Event(E):

In **probability** theory, an **event** is a set of outcomes of an experiment (a subset of the sample space).

Example

➢ Rolling a dice is a random experiment.

Here the number of possible outcomes 6.

Sample space = $\{1, 2, 3, 4, 5, 6\}$

Getting an even number is an event.

$$\textbf{Probability of an event} = \frac{\text{total no of elments in the event set } (n(E))}{\text{total no.of elements in the sample space} (n(S))}$$

❖ Points to remember

- $P(S) = 1$
- $0 \leq P(E) \leq 1$

Solved examples:

1. Find the probability of getting an odd no. when a dice is thrown?

Solution:

Here $S = \{1,2,3,4,5,6\}$

$E = \{1,3,5\}$

P (getting an odd number)= $\frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

2. A coin is tossed twice .find the probability of getting at least one head.

Solution:

Here $S = \{HH, HT, TH, TT\}$

$E = \{HT, TH, HH\}$

P (getting at least one head)= $\frac{n(E)}{n(S)} = \frac{3}{4}$

3. A number a is chosen at random from the numbers 4, 6, 1, 92, 32, 56, 98, 11, 55. What is the probability that a < 50.

Solution:

Here $S = \{4, 6, 1, 92, 32, 56, 98, 11, 55\}$

$E = \{4, 6, 1, 32, 11\}$

P (choosing a < 50)= $\frac{n(E)}{n(S)} = \frac{5}{9}$.

4. Two dice are thrown simultaneously. Find the probability of getting a sum greater than or equal to 8 on adding the two faces.

Solution:

Here S ={(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)

(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)

(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)

(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)

(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)

(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)}

E = {(2,6)(3,5)(3,6)(4,4)(4,5)(4,6)(5,3)(5,4)(5,5)(5,6)(6,2)(6,3)(6,4)(6,5)(6,6) }

P (getting a sum greater than or equal to 8) = $\frac{n(E)}{n(S)} = \frac{15}{36}$

5. Tickets numbered 1 to 25 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is multiple of 3 or 5?

Solution:

Here S = {1, 2, 3,…………, 24, 25}

E = Event of getting a multiple of 3 or 5 = {3, 6, 9, 12, 15, 18, 21, 24, 5, 10, 20, 25}

$P(E) = \frac{n(E)}{n(S)} = \frac{12}{25}$.

II. LOGICAL REASONING

Unit - 1 : Analogy basing on kinds of relationships, Simple Analogy; Pattern and Series of Numbers, Letters, Figures. Coding-Decoding of Numbers, Letters, Symbols (Figures), Blood relations

1. <u>Analogy</u>

Definition : An analogy compares the relationship between two things or ideas to highlight some point of similarity. It is a way to clarify an idea or an unfamiliar concept by comparing it to something familiar.

Look at this example:

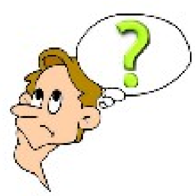
appalling : pleasing :: interesting : ____________

A. amusing

B. engaging

C. enthralling

D. boring



NOTE :

- The example given above asks you to identify the relationship between pairs of words.
- To answer this question, you must first decode the symbols. The colon (:) stands for the phrase "is related to," and the double colon (::) can be read as "in the same way that." Thus, you would read the above example like this: "Appalling is to pleasing in the same way that interesting is to…"
- To figure out the missing word, you need to identify the relationship between the first two elements precisely as possible, and choose a word that will make the final pair have a parallel relationship.
- Most accurately, you might describe the relationship between appalling and pleasing as "appalling is an antonym of pleasing."
- Now read the first word of the second pair, supplying the same relationship: "interesting is an antonym of…"
- The only word that fits this blank adequately is "boring," the opposite of 'interesting'. The other three options do not fit the blank as they are the synonyms of interesting.
- In the example above, the relationship between the words in the first pair is compared to the relationship of words in the second pair. This is what we call an analogy is.

For example - **butterflies : swarm :: fish : school**


You can read this analogy as:

Butterflies are to swarm as fishes are to school.

In this example, swarm is the specific term for a group of butterflies. Similarly, school is the specific term for a group of fish.

Both the pairs of words in the analogy illustrate the same relationship.

KINDS OF RELATIONSHIPS

1. Instrument and measurement

- Ex-Barometer : Pressure - Barometer is an instrument used to measure pressure.
- Some more examples -
- Thermometer : Temperature
- Odometer : Speed
- Scale : Length
- Balance :Weight
- Rain gauge : Rain

2. Quantity and Unit -

- Ex- Length : Metre - Metre is the commonly used unit of length.
- Some more examples –
- Mass : Kilogram
- Force : Newton
- Volume : Litre
- Time : Hour
- Temperature : Degrees

3. Individual and group -

- Ex – Sailors : Crew - A group of sailors is called a crew.
- Some more examples –
- Cattle : Herd
- Flowers : Bouquet
- Grapes : Bunch
- Singer : Chorus
- Man : Crowd

4. Animal and young one –

- Ex – Horse : Pony - Pony is the young one of horse.
- Some more examples –
- Cat : Kitten
- Sheep : Lamb
- Cow : Calf
- Dog : Puppy
- Man : Child

5. Male and female –

- Ex- Horse : Mare - Mare is the female horse.
- Some more examples –
- Dog : Bitch
- Son : Daughter
- Lion : Lioness
- Gentleman : Lady
- Nephew : Niece

6. Individual and class –

- Ex – Lizard : Reptile - Lizard belongs to the class of reptiles.
- Some more examples
- Man : Mammal
- Ostrich : Bird
- Butterfly : Insect
- Snake : Reptile
- Whale : Mammal

7. Individual and dwelling places

- Ex – Dog : Kennel - A dog lives in a kennel.
- Some more examples –
- Bee : Apiary
- Cattle : Shed
- Lion : Den
- Poultry : Farm
- Fish : Aquarium

8. Study and topics –

- Ex – Ornithology : Birds - Ornithology is the study of birds
- Some more examples –
- Botany : Plants
- Entomology : Insects
- Zoology : Animals
- Oology : Eggs
- Virology : Viruses

9. Worker and tool –

- Ex – Carpenter : Saw - Saw is a tool used by the carpenter.
- Some more examples –
- Woodcutter : axe
- Tailor : needle
- Soldier : gun
- Doctor : stethoscope
- Farmer : plough

10. Tool and action –

- Ex- Needle : Sew - A needle is used for sewing
- Some more examples –
- Knife : cut
- Pen : write
- Spoon : feed
- Gun : shoot

- Axe : grind

11. Worker and working place-

- Ex – Chef : Kitchen - A chef works in a kitchen
- Some more examples –
- Farmer : field
- Warrior : battlefield
- Teacher : school
- Doctor : hospital
- Clerk : office

12. Worker and product –

- Ex – Mason : Wall - A mason builds a wall.
- Some more examples –
- Farmer : crop
- Hunter : prey
- Carpenter : furniture
- Author : book
- Butcher : meat

13. Product and Raw Material-

- Ex-Prism: Glass- Prism is made of glass.
- Some more examples –
- Butter : Milk
- Wall : Brick
- Furniture : Wood
- Shoes : Leather
- Oil : Seed

14. Part and whole relationship –

- Ex- Pen : Nib - Nib is a part of a pen
- Some more examples –
- Pencil : lead
- House : keychain

- Fan : blade
- Class : student
- Room : Window

15. Word and Intensity -

- Ex- Anger: Rage- Rage is of higher intensity than anger.
- Some more examples –
- Wish: Desire
- Kindle: Burn
- Sink: Drown
- Quarrel: War
- Error: Blunder

16. Word and Synonym –

- Ex- Abode : Dwelling – Abode means the same as dwelling. Thus, dwelling is the synonym of abode.
- Some more examples –
- Ban : Prohibition
- Assign : Allot
- Vacant : Empty
- House : Home
- Flaw : Defect

17. Word and Antonym-

- Ex- Attack : Defend – Defend means the opposite of attack. Thus, Defend is the antonym of Attack
- Some more examples –
- Advance : Retreat
- Cruel : kind
- Best : Worst
- Fresh : Stale
- Ignore : Notice

Examples –

In the following questions, find out the RELATION between given two words in capital letter and pick up one word proportionately from the options that bear the same relation.

1. Day: Night **::________: ________**
   (1) Half : Full (2) Tall : Fat
   (3) East : West (4) Food : Vegetable

Answer - East : West (Opposite To Each Other).

2. Distance: Mile **::________ : ________**
   (1) Weight : Scale (2) Fame : Television
   (3) Field : Plough (4) Liquid: Litre

Answer - Liquid : Litre (2^{nd} one is the unit of 1^{st} one).

3. Ring: Finger :: Shoe : ________
   (1) Socks (2) Case
   (3) Foot (4) Market

Answer - Foot (2^{nd} one is the body part in which the 1^{st} one is worn)

4. Court: Justice :: School **:** ________
   (1) Teacher (2) Education
   (3) Student (4) Discipline

Answer **-** Education (The thing that is imparted in the institution).

5. Mouse: Cat :: Worm **:** ________
   (1) Trap (2) Bird
   (3) Paw (4) Grab

Answer - Bird (Prey : Predator)

2. SERIES

- **NUMBER SERIES** –
  A series of numbers which follow a certain pattern throughout.

Case I –
Finding the difference between the terms in the given series –

Ex 1 : Which number would replace ‘ ? ‘ in the series : 7 , 12 , 19 , ? , 39
a) 29 b) 28 c) 26 d) 24

Solution - Difference between 7 and 12 = 5

Difference between 12 and 19 = 7

Note – Here the difference increases by 2 , so the next difference should be 9.

Thus, answer = 19 + 9 = 28.

Ex 2 : Which is the number that comes next in the sequence : 0 , 6 , 24 , 60 , 120 , 210 ?

a) 240 b) 290 c) 336 d) 504

Solution – Pattern of the series ; $1^3 - 1$, $2^3 - 2$, $3^3 - 3$, $4^3 - 4$, 5^3 - 5 , 6^3 - 6

Next number $7^3 - 7 = 343 - 7 = 336$

Hence, the answer is 336.

Ex 3 : Which is the number that comes next in the following sequence : 4 , 6 , 12 , 14 , 28 , 30 , ?

a) 32 b) 60 c) 62 d) 64

Solution – The given sequence is the series 4, 4+2 (=6), 6*2 (=12), 12+2, 14*2, 28+2…

So pattern followed is +2,*2,+2,*2, …..

So, next number is 30*2

Hence, answer = 60

Ex 4 : Find out the missing number in the following sequence : 1 , 3 , 7 , ? , 21

a) 10 b) 11 c) 12 d) 13

Solution - Pattern followed is + 2 , + 4 , …..

Missing number = 7 + 6 = 13

Hence, answer = 13.

Ex 5 : Which fraction comes in the sequence $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}$, ?

a) $\frac{9}{32}$ b) $\frac{10}{17}$ c) $\frac{11}{34}$ d) $\frac{12}{35}$

Solution- The numerators of the fractions in the series have a difference of 2. The denominators of the fractions form the series 2,4,8,16 i.e 2^1, 2^2, 2^3, 2^4…So the numerator of the next fraction will be 7+2=9 and denominator will be 2^5=32.

Answer = $\frac{9}{32}$

Elementary Idea of Progressions

1. **Arithmetic Progression (A.P)-** The progression of the form a, a+d, a+2d…..is known as AP with first term= a and common difference = d

   Ex 1- 3, 6, 9, 12…… is an AP with a=3, d=6-3=3

   In an AP we have n*th* term= a + (n-1)d

   Ex 2- In the series 357, 363,369,….,What will be the 10^{th} term ?

   a) 405 b) 411 c) 413 d) 417

   Solution- The given series is an A.P in which a=357, d=6

   10^{th} term = a+(10-1)d

   = a+9d

   = (357+9 × 6)

   = 357+54

   = 411

   Ex3- How many terms are there in the series 201, 208, 215, …, 369 ?

   A) 23 b) 24 c) 25 d)26

   Solution- The given series in an AP in which a=201, d=7

   Let the number of terms be n.

   369=201+ (n-1) × 7

   $\Rightarrow$ 369=201+7n-7

   $\Rightarrow$ 168=7n-7

   $\Rightarrow$ 7n= 175

   $\Rightarrow$ n = 25

   Answer = 25

2. **Geometric Progression (G.P)-** The progression of the form a,ar,ar^2, ar^3,…..is known as GP with first term = a and common ratio= r

   Ex 1- 1,5, 25, 125, …. is a GP with a = 1 and r = $\frac{5}{1} = \frac{25}{5}$ =………= 5

   In a GP we have n*th* term= ar^{n-1}

   Ex 2- In the series 7,14,28,…. What will be the 10^{th} term?

   a) 1792 b) 2456 c) 3584 d) 4096

   Solution- Clearly, 7*2=14, 14*2=28,…..and so on

   In the given series of GP a=7, r =2

   10^{th} term = $ar^{(10-1)}$

   = ar^9

   = 7×2^9

   =7× 512

   = 3584

Answer= 3584

Ex 3- Find the number of terns in GP 6, 12 ,24, ... , 1536 ?

a) 7 b)9 c)8 d)10

Solution- $a_1 = 6$ $a_2 = 12$ $a_n = 1536$

$r = \frac{a2}{a1} = \frac{12}{6} = 2$

Now we have, $1536 = ar^{n-1}$

⇒ $1536 = 6 \times 2^{n-1}$

⇒ $256 = 2^{n-1}$

⇒ $2^8 = 2^{n-1}$

⇒ $8 = n-1$

⇒ $n = 9$.

➢ <u>LETTER SERIES</u>

In this series, only letters are available which follow a certain pattern throughout.

<u>Position of Letters</u>

1	2	3	4	5	6	7	8	9	10	11	12	13
↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕
A	B	C	D	E	F	G	H	I	J	K	L	M
26	25	24	23	22	21	20	19	18	17	16	15	14
↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕
Z	Y	X	W	V	U	T	S	R	Q	P	O	N

<u>Quick tricks :</u>

1. Starting point of the series is called left end and end point of the series is called right end.

2. To solve the question easily we should break the series in combination of 5-5 elements – ABCDE / FGHIJ / KLMNO / PQRST / UVWXY / Z - it will help in counting the letters.

3. There are some key words which help in remembering the place values of the letters. Once the candidate knows the position of alphabets, he requires to learn time management.

TABLE OF 3

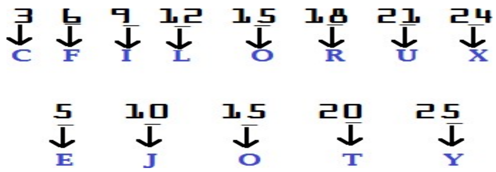


TABLE OF 5

Words given above are the arrangement of alphabets having position multiples of 3 in first line and 5 in second line.

4. The opposite letters

A ↔ Z (**AZ**ad)	F ↔ U (**FU**ll)	K ↔ P (**K**an**P**ur)
B ↔ Y (**B**o**Y**)	G ↔ T (**G T** road)	L ↔ O (**LO**ve)
C ↔ X (**C**ru**X**)	H ↔ S (**H**igh **S**chool)	M ↔ N (**M**a**N**)
D ↔ W (**D**e**W**)	I ↔ R (**I**ndian **R**ailway)	
E ↔ V (**EV**en)	J ↔ Q (**J**ungle **Q**ueen)	

Examples –

1. Solve the series : JAF, JEF, JIF, JOF, ?

a) PIG b) PET c) JUF d) POT

Solution –

The middle letters which are vowels have an increasing trend of A, E, I, O, U and remaining letters have been retained as it is. So answer would be **JUF.**

2. Solve the series - WXCD, UVEF, STGH, QRIJ?

a) OPKL b) AYBZ c) JIRQ d) LRMS

Solution –

The last two letters of every word is in ascending order and the first two letters are in descending order.

OP precedes QR and KL succeeds IJ.

Answer - **OPKL**

3. In the word EMASCULATE let the place value of the letters according to English alphabets be written in descending order then which number is 4th from the left end?

a. 13 b.14 c.12 d.1

Solution –

E M A S C U L A T E

5 13 1 19 3 21 12 1 20 5

Descending order -

21 20 19 **13** 12 5 5 3 1 1

So, answer is 13 , 4^{th} from left end.

4. If only each of the vowels in the word IMPOSE is changed to the next letter in the English alphabet then which of the following will be the fifth letter from the left end ?

a) P b) J c) F d) S

Solution -

I M P O S E

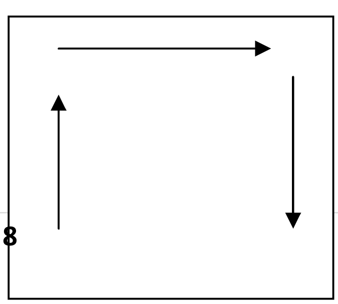
J M P P **S** F

So, the answer is S

- **FIGURE SERIES**

Definition – In a figure series, there is a sequence of figures depicting a change step by step. Either one of these figures is out of order and has to be omitted or figure has to be selected from a separate set of figures which would continue the series.

There are two directions mostly used in the figures – i) clockwise direction ii) anti – clockwise direction. A clockwise direction movement will be as in a square boundary.



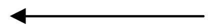


Fig - A clockwise direction movement in a square boundary.

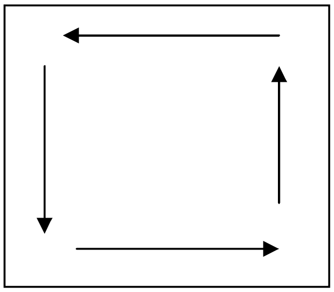


Fig – An anticlockwise direction movement in a square boundary.

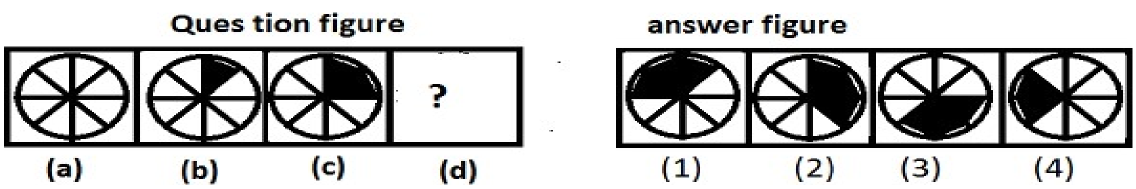
Note – In a square boundary means a square box.

<u>Four Figure series</u> –

In this case , the series or sequence is indicated by four problem figures and it is required to select a figure from amongst the answer figures which would be fifth figure to continue the series.

Examples –

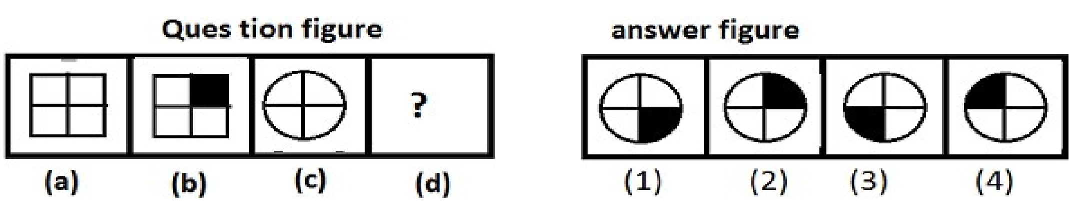
1. Find the figure that will replace the question mark



Answer - Option (2)

The sequence of this series is that the circle has been divided into various sectors which are getting shaded in clockwise direction by adding a cord each time.

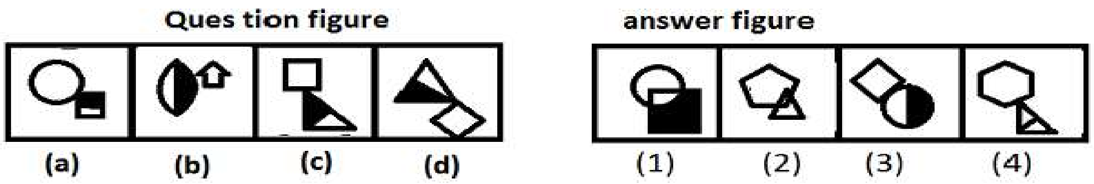
2. Which figure can be placed in place of question mark?



Answer − Option (2)

Figure (a) and (b) are related to each other by getting shade of upper right side quarter therefore in this way figure (2) will make pair with figure (c).

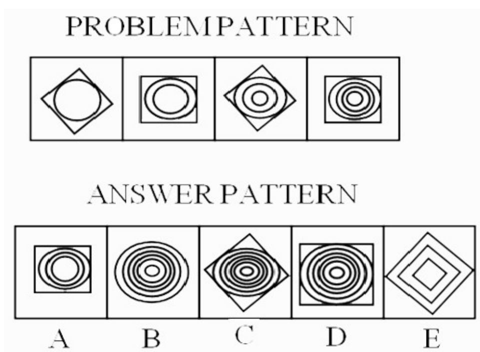
3. Find the similar figure as that of the question figure.



Answer − Figure (3)

In question figure all blocks consist of two figures touching each other and half of any of them is shaded. The sequence in the question figure also shows that in the odd number blocks, top figure is empty and bottom is shaded while in the even number blocks top figure is shaded while the bottom one is empty. Hence figure (3) is same as question figure.

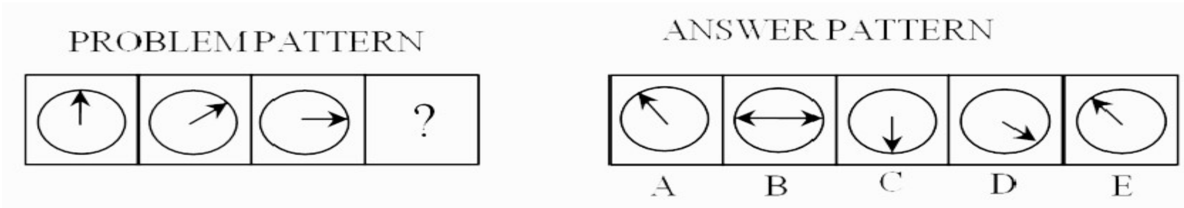
4. If the figures continue to change in the same order what should the fifth figure be?"



Solution: You will see two different things happening here. The number of circles is increasing 1, 2, 3, 4, so that the next figure would have 5 circles. But note that the square is also turning. First the point is up, then the flat side, then again the point is up, and then again the flat side. In the next place, therefore, the point should be up. But in figure C there are 6 circles inside the rectangle.

Answer: So none of the figure is correct.

5. Continue the series :



Answer – D .

Sol: Here the arrow in the circle goes on rotating in clockwise direction at an angle of 45° each time. Hence the answer would be (d).

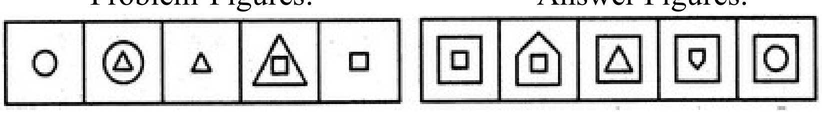
Five figure series:

In this case , the series or sequence is indicated by five problem figures and it is required to select a figure from amongst the answer figures which would be sixth figure to continue the series.

Examples –

1. Select a figure from amongst the answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:



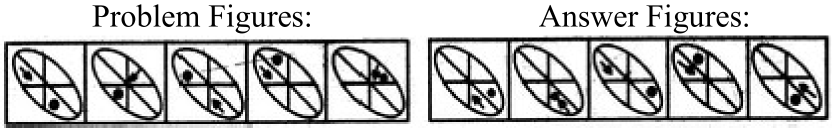
(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

Answer: Option **(4)**

In one step, the existing element enlarges and a new element appears inside this element. In the next step, the outer element is lost.

2. Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:



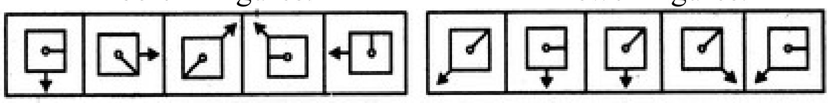
(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

Answer : Option **(1)**

In each step, the dot moves one space clockwise and the arrow moves two spaces clockwise.

3. Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:



(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

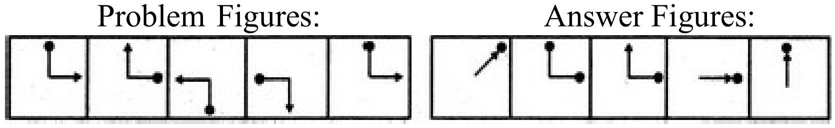
Answer - Option **(3)**

The pin rotates 45^o clockwise and 90^o clockwise alternately and moves one space (each space is equal to half-a-side of the square) and two spaces clockwise alternately.

The arrow rotates 90^o anti clockwise and 45^o anti clockwise alternately and moves two spaces and one space.

4. Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:



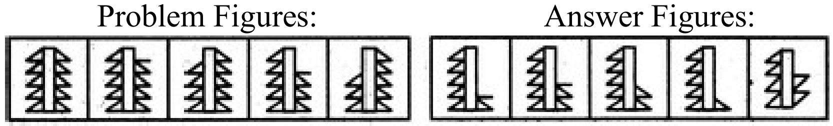
(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

Answer - Option **(3)**

In each step, the pin rotates 90^o clockwise and the arrow rotates 90^o anti clockwise.

5. Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:



(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

Answer – Option **(2)**

In one step, the figure gets laterally inverted and one line segment is lost from the upper end of the RHS portion of the figure. In the next step, the figure gets laterally inverted and one line segment is lost from the upper end of the LHS portion of the figure.

\

3. CODING & DECODING

- **Coding** is a process in which a word, a number, or a series of combination of words and numbers is expressed in a particular code or pattern based on various rules. You have to answer the questions based on these set of rules.
- **Decoding** is the process of (interpreting) deciphering the coded pattern and reverting it to its original form from the given codes. Hence, you are required to understand the logic behind the coding pattern and then apply this logic to find answers.

LETTER CODING:

Here letters are assigned codes according to a set pattern or rule concerning the movement or reordering / rearranging of letters and you need to detect this rule to decode a massage. Sometimes, specific codes are assigned to particular letters without any set pattern. Observe a few examples to know the various reordering or rearranging techniques.

Ex-1

In a code language if TRAINS is coded as RTIASN, how PISTOL will be coded in the same language?
(a) SITLOP
(b) IPSTLO
(c) SIPTLO
(d) IPTSLO

Solution: (Answer – d)

If we compare the basic word {TRAINS} with the coded word {RTIASN}, we would see that the letters used in the word are same as in the basic word but their order of placement has been changed. Letter T at first position of the basic word has been placed at second position in the coded word and the letter R at the second position has been placed in the first position.
It means that in this question, letters of the basic word have been interchanged i.e. first letter with second, third with the fourth and so on. And thus we get the coded word. So PISTOL will be coded as IPTSLO. Hence option (d) is the answer.

Ex-2

In a certain code, TEACHER is written as VGCEJGT. How is CHILDREN written in that code?
(a) EJKNEGTP (b) EGKNFITP (c) EJKNFGTO (d) EJKNFTGP

Solution: (Answer- d)

Each alphabet in the word “TEACHER " is moved two steps forward to obtain the corresponding alphabet of the code.
T E A C H E R

V G C E J G T
(Each alphabet is increasing by 2)
Similarly, we have
C H I L D R E N
E J K N F T G P

Ex-3

In a certain code language, the word ROAD is written as WTFI. Following the same rule of coding, what should be the word for the code GJFY?

(a) REAP (b) TAKE (c) BEAT (d) LATE

Solution: (Answer- c)

Each alphabet of the word is five steps behind the corresponding alphabet of the given code word.

Hence, BEAT is coded as GJFY.

Ex-4

If 'tee see pee' means 'drink fruit juice'; 'see kee lee' means 'juice is sweet' and 'lee ree mee' means 'he is intelligent', which word in that language means 'sweet'?
(a) see
(b) kee
(c) lee
(d) pee

Solution: (Answer b)
In the first and second statement, the common word is 'juice' and the common code word is 'see'. So, 'see' means 'juice'.
In the second and third statements, the common word is 'is' and the common code is 'lee'. So 'lee' means 'is'. Thus in the second statement, the remaining word 'sweet' is coded as 'kee'. Hence the answer is choice (b).

NUMBER CODING:

Numerical code is given or value is assigned to a word. Here the only way to relate the alphabets & numbers is by associating the positions of the letters in the English alphabet. Sometimes any mathematical operation like addition or subtraction can be performed using the position of the letters. Direct coding questions can also be asked.

Ex-1 If PAINT is coded as 74128 and EXCEL is coded as 93596, then how would you encode ACCEPT?

(a) 455978
(b) 547978
(c) 554978
(d) 735961

Solution: (Answer- a)
In the given code the alphabets have been coded as follows:

P	A	I	N	T	E	X	C E L
7	4	1	2	8	9	3	5 9 6

So, in ACCEPT, A is coded as 4, C as 5, E as 9, P as 7 and T as 8. Hence the correct code is 455978 and therefore the answer is Choice (a).

Ex-2 If DELHI is coded as 73541 and CALCUTTA as 82589662, how can CALICUT be coded?

a. 5279431
b. 5978213
c. 8251896
d. 8543691

Solution.(Answer: c)
`The alphabets are coded as follows :

D	E	L	H	I	C	A	U	T
7	3	5	4	1	8	2	9	6

So, in CALICUT,
C is coded as 8,
A as 2,
L as 5,
I as 1,
U as 9 and
T as 6.
Thus, the code for CALICUT is 8251896.

Ex-3 If in a certain code, TWENTY is written as 863985 and ELEVEN is written as 323039, how is TWELVE written in that code ?

a) 863203
b) 863584
c) 863903
d) 863063

Solution (Answer: a)
The alphabets are coded as shown :

T	W	E	N	Y	L	V
8	6	3	9	5	2	0

So, In TWELVE ,
T is coded as 8,
W as 6,
E as 3,
L as 2,
V as 0.
Thus, the code for TWELVE is 863203.

Ex-4 In a certain code 'MISSIONS' is written as 'MSIISNOS'. How is 'ONLINE' written in that code?

1. OLNNIE

2. ONILEN

3. NOILEN

4. LNOENI

5. ONNLIE

Solution: Answer - Option 1

Explanation: First and last letter remain same. The others interchange their positions in pair of two.

So, NL become LN and IN become NI so code of ONLINE will be OLNNIE.

SYMBOL CODING

Directions (Q1-5): In each of the following questions, there is a group of letters followed by four combinations of digits/symbols (A), (B), (C) and (D). You have to find out which of the combinations correctly represents the group of letters based on the following digits/symbol coding system and the conditions those follow and mark the number of that combination as the answer. If none of the combinations is correctly represents the group of letters then mark (E).

Letter:	L	H	U	B	E	P	N	A	K	I	R	S	T	M	V
Digits/Symbols:	5	<	4	#	@	3	*	^	8	%	9	!	~	1	$

Conditions:

i) If the first letter is a vowel and the last letter is a consonant, then the codes are to be interchanged.
ii) If both the first and the last letters are vowels, then both are to be coded as the code for the last letter
iii) If any word has more than two vowels, then all vowels are coded to be as the code for I.

UNRBV

A. 4*9#$
B. $*9#$
C. $*9#4
D. $*4#9
E. None of these.

Ans. C

Solution: UNRBV - $*9#4 (Condition i)

SMALKI

A. %15!8^
B. !1^58%
C. 1#!568
D. !1^5%8
E. None of these
Ans . B
Solution: SMALKI - !1^58%

AMBLPU

A. ^5#34^
B. 1#543^
C. ^1#534
D. 41#534
E. None of these

Ans.D

Solution: AMBLPU - 41#534 (Condition ii)

KINAHE

A. 9*#%15
B. 8%*%<%
C. @*%$56
D. %^85@1
E. None of these
Ans.B

Solution: KINAHE - 8%*%<% (Condition iii)

EMKLVP

A. $5&58^
B. $5&^85
C. @1853$
D. 3185$@
E. None of these
Ans.D

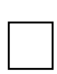
Solution: EMKLVP - 3185$@ (Condition i)

4. BLOOD RELATIONS

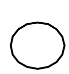
Definition- A person who is related to another by birth rather than by marriage.

NOTE- Relation on the mother side is called **maternal** and that on the father side is called **paternal** and if the relation is on the partner side (Husband or wife) is called **in-law.**

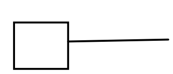
A method known as **FAMILY TREE** is used to solve the questions regarding blood relation which is just a graphical method to show all the possible relation.



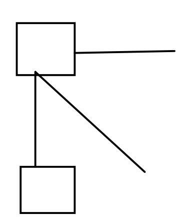
Used for males



Used for females



Husband wife relationship



Son and daughter

Indirect Reference	The real relation
Father’s or Mother’s Daughter	Sister
Father’s or Mother’s Son	Brother
Father’s or Mother’s Sister	Aunt
Father’s or Mother’s Brother	Uncle
Father’s or Mother’s Mother	Grandmother
Father’s or Mother’s Father	Grandfather
Daughter’s Husband	Son-in-law
Son’s Wife	Daughter – in – law
Husband’s or Wife’s Brother	Brother – in – law
Husband’s or Wife’s Sister	Sister – In – law
Brother’s Daughter	Niece
Brother’s Son	Nephew
Brother’s Wife	Sister-in-law
Sister’s Husband	Brother- in- law
Aunt’s or Uncle’s Son or Daughter	Cousin
Granddaughter’s or Grandson’s daughter	Great grand daughter

The first step to being able to solve blood relation questions is a logical reasoning of the relations that exist between different members of a family, both close and far.

EXAMPLES

Question 1: Pointing to a girl in the photograph, Ajay said, "Her mother's brother is the only son of my mother's father." How is the girl's mother related to Ajay ?

A) Mother

B) Sister

C) Aunt

D) Grandmother

E) None of these

Solution:

Only son of Ajay's mother's father -- Ajay's maternal uncle.

So, the girl's maternal uncle is Ajay's maternal uncle.

Thus, the girl's mother is Ajay's aunt.

Question 2:

1. A + B means A is the brother of B
2. A × B means A is the father of B
3. A ÷ B means A is the mother of B

Which of the following would mean "G is the son of H"?

A)H ×I × G

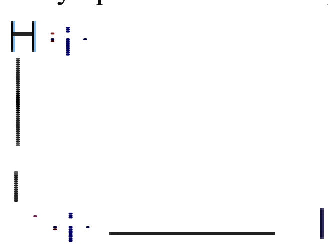
B)H + G × I

C)H ÷ G ÷ I

D) H × G + I

Solution: Answer: Option D

Go by options. In fourth option, our diagram will be like



We don't know the gender of I. So. We will not put any symbol on its side.

Question 3.A is B's sister. C is B's mother. D is C's father. E is D's mother. Then, how is A related to D?

A. Grandfather

B. Grandmother

C. Daughter

D. Granddaughter

Answer: D) Granddaughter

Explanation:

A is the sister of B and B is the daughter of C.

So, A is the daughter of C. Also, D is the father of C.

So, A is the granddaughter of D.

Question 4.P is the brother of Q and R. S is R's mother. T is P's father. Which of the following statements cannot be definitely true ?

A. T is Q's father

B. S is P's mother

C. P is S's son

D. Q is T's son

Answer: D) Q is T's son

Explanation:

P, Q, R are children of same parents. So. S who is R's mother and T, who is R's father will be mother and father of all three.

However, it is not mentioned whether Q is male or female So, D cannot be definitely true.

Question 5.Pointing to a person, a man said to a woman, "His mother is the only daughter of your father." How was the woman related to the person ?

A. Aunt

B. Mother

C. Wife

D. Daughter

Answer: A) Aunt

Explanation:

Daughter of your father — your sister. So, the person's mother is woman's sister or the woman is person's aunt.

UNIT – 2 : Logical Statements – Two premise argument, More than two premise argument using connectives

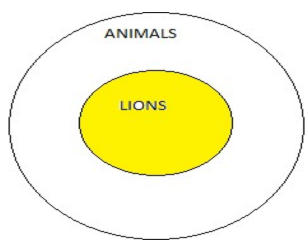
In logic, any statement is termed as the Proposition. Thus a proposition is a statement expressing certain relation between two or more terms, analogous to a sentence in grammar.

The proposition consists of three parts:

- Subject-The subject is that about which something is said
- Predicate-The predicate is the part of the proposition denoting that which is affirmed or denied about the subject.
- Copula-The copula is that part of the proposition which denotes the relation between the subject and the predicate.
- Consider the proposition('Man is cultured')
  Here 'man' is the subject.
  'Cultured' is the quality affirmed for this subject. So it is the predicate.
  'is' denotes the relation between the subject and the predicate. So it is the copula.

FOUR FOLD CLASSIFICATION OF PROPOSITIONS:

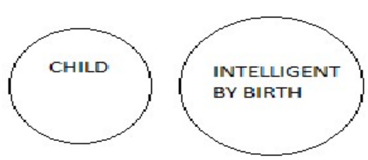
a. **Universal Affirmative Proposition**
   Example-All lions are animals.



From this, we cannot say 'All animals are lions.

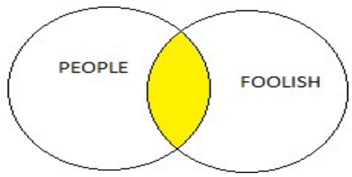
b. **Universal Negative Proposition**

   Ex. - No child is intelligent by birth.



c. Particular Affirmative Proposition

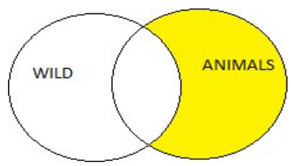
Ex. - Some people are foolish.



Here the subject term 'Some People' is used not for all but only for some people and similarly the predicate term 'Foolish' is affirmed for a part of the subject.

d. Particular Negative Proposition

e.g. " Some animals are not wild "



Here the subject term ' some animals' is used only for a part of its class while the predicate term 'wild' is not denied in entirety to the subject term.

These facts can be summarized as follows :

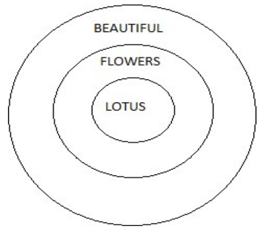
Proposition	Type
a) Universal Affirmative Proposition	All S is P.
b) Universal Negative Proposition	No S is P.
c) Particular Affirmative Proposition	Some S is P.
d) Particular Negative Proposition	Some S is not P.

SYLLOGISM:

Two premise argument

In Syllogism, a conclusion has to be drawn from two propositions , referred to as the Premises.

Example 1– a. All lotus are flowers.
b. All flowers are beautiful.
c. All lotus are beautiful.



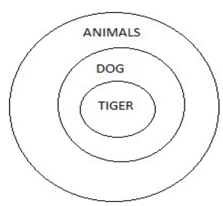
The propositions a) and b) are the Premises and the proposition c) is called the Conclusion which follows from the first two propositions.

Syllogism is concerned with 3 terms –

1. **Major term** : It is the predicate of the conclusion and is denoted by P .
2. **Minor term** : It is the subject of the conclusion and is denoted by S.
3. **Middle term** : It is the term common to both the premises and is denoted by M.

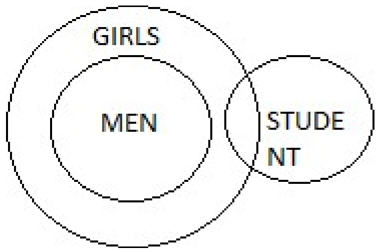
Example 2- Premises : 1. All dogs are animals.
2. Tiger is a dog.

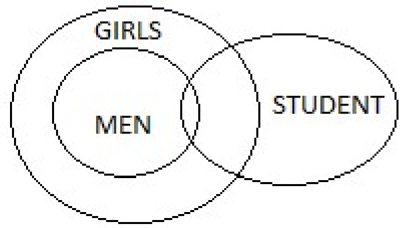
Conclusion : Tiger is an animal.



Rules for deriving the conclusion :

I. The conclusion contain the middle term.
Example - Statements : 1. All men are girls.
2. Some girls are students.
Conclusion : 1. All girls are men.
2. Some students are girls.





A B

Here both the conclusions 1 and 2 contain the middle term ‘girls’. From the above figures, It is clear that conclusion 1 does not follow, but conclusion 2 follows.

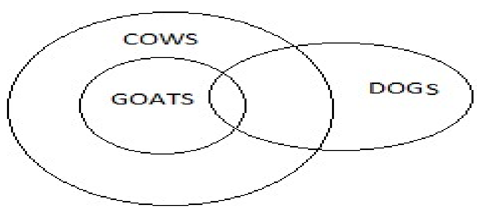
II. No term can be distributed in the conclusion unless it is distributed in the premises.

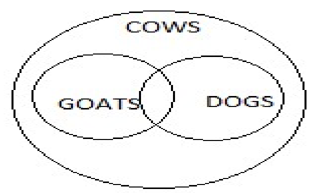
Example - Statements : 1. Some dogs are goats.

2. All goats are cows.

Conclusions : 1. All cows are goats.

2.Some dogs are cows.





A B

Statement 1 is an I type proposition which distributes neither the subject nor the predicate . Statement 2 is an A type proposition which distributes the subject i.e ‘goats’ only.

Conclusion 1 is an A type proposition which distributes the subject ‘cow’ only.

Since the term ‘cow’ is distributed in conclusion 1 without being distributed in the premises , so conclusion 1 cannot follow.

III. The middle term (M) should be distributed at least once in the premises. Otherwise, the conclusion cannot follow.

For the middle term to be distributed in a premise,

i) M must be the Subject if premise is an A proposition.

ii) M must be Subject or Predicate if premise is an E proposition.

iii) M must be Predicate if premise is an O proposition.

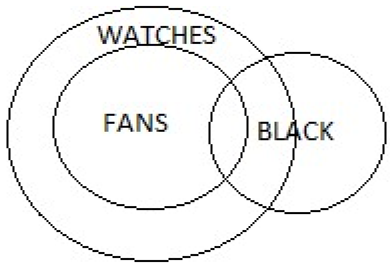
NOTE : In an I proposition, which distributes neither the subject nor the Predicate, the middle term cannot be distributed.

Example : Statements : 1. All fans are watches.

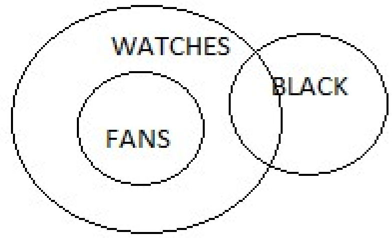
2.Some watches are black.

Conclusions : 1. All watches are fans.

2. Some fans are black.



A



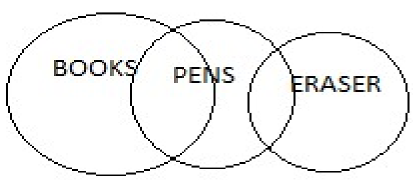
B

In the premises, the middle term is ‘watches’. It is not distributed in the first premise which is an A proposition as it doesn’t form its subject. Also, it isn’t distributed in the second premise which is an I proposition. Since the middle term is not distributed at least once in the premises, so no conclusion follows.

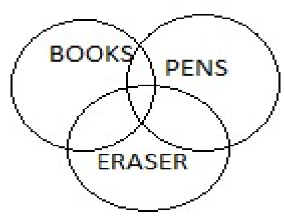
IV. No conclusion follows

a) If both the premises are particular

Example : Statements : 1. Some books are pens.
2.Some pens are
erasers. Conclusions : 1. All
books are erasers.
2.Some erasers are books.



A



B

Since both the premises are particular, no conclusion follows.

b) If both the premises are negative

Example : Statements : 1. No flower is mango.
2.No mango is cherry.
Conclusions : 1. No flower is cherry.
2.Some cherries are mangoes.

CORE PAPER - XI

MULTIVARIATE CALCULUS

Objective: The objective of this course to introduce functions of several variable to a student after he has taken a course in one variable calculus. The course will introduce partial derivatives and several of its consequences and will introduce double and triple integrals along with line integrals which are fundamental to all streams where calculus can be used.

Expected Outcomes: After reading this course a student will be able to calculate partial derivatives,directional derivatives,extremum values and can calculate double, triple and line integrals. He will have idea of basic vector calculus including green's theorem, divergence theorem.and stokes theorem. He can take courses in calculus on manifolds, Differential geometry and can help in numerical computations involving several variables.

UNIT-I

Functions of several variables, limit and continuity of functions of two variables. Partial differentiation, total differentiability and differentiability, sufficient condition for differentiability. Chain rule for one and two independent parameters, directional derivatives, the gradient, maximal and normal property of the gradient, tangent planes.

UNIT-II

Extrema of functions of two variables, method of Lagrange multipliers, constrained optimization problems.

Definition of vector field, divergence and curl, Double integration over rectangular region, double integration over nonrectangular region. Double integrals in polar co-ordinates,

UNIT-III

Triple integrals, Triple integral over a parallelepiped and solid regions. Volume by triple integrals, cylindrical and spherical co- ordinates. Change of variables in double integrals and triple integrals.

UNIT-IV

Line integrals, Applications of line integrals: Mass and Work. Fundamental theorem for line integrals, conservativevector fields, independence of path. Green's theorem, surface integrals, integrals over parametrically defined surfaces. Stokes' theorem, The Divergence theorem.

BOOKS RECOMMENDED:

1. M. J,.Strauss, G. L. Bradley and K. J. Smith, Calculus (3rd Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.
2. S C Mallik and S Arora: Mathematical Analysis, New Age International Publications

BOOK FOR REFERENCES:

1. G.B. Thomas and R.L. Finney, *Calculus,* 9th Ed., Pearson Education, Delhi, 2005.
2. E. Marsden, A.J. Tromba and A. Weinstein, *Basic Multivariable Calculus,*Springer(SIE). Indian reprint, 2005.

3. James Stewart, *Multivariable Calculus, Concepts and Contexts,* 2^{nd} Ed., Brooks/*Cole,* Thomson Learning, USA, 2001.
4. S Ghorpade, B V Limaye, Multivariable calculus, Springer international edition

CORE PAPER –XII
LINEAR ALGEBRA

Objective: Linear algebra is a basic course in almost all branches of science. A full course in undergraduate program will help students in finding real life applications later.. The objective of this course is to introduce a student the basics of linear algebra and some of its application

Expected Outcomes: The student will use this knowledge wherever he/She goes after undergraduate program. It has applications in computer science, finance mathematics, industrial mathematics, bio mathematics and what not.

UNIT-I

Vector spaces, subspaces, examples, algebra of subs paces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces. Linear transformations, null space, range, rank and nullity of a linear transformation.

UNIT-II

Matrix representation of a linear transformation, Algebra of linear transformations, Isomorphisms, Isomorphism theorems, invertibility and isomorphisms, change of coordinate matrix, Dual spaces, dual basis, double dual, transpose of a linear transformation and its matrix in the dual basis, annihilators, Basics of Fields.

UNIT-III

Eigenspaces of a linear operator, diagonalizability. Invariant subspaces and Cayley-Hamilton theorem, the minimal polynomial for a linear operator, Inner product spaces and norms, Gram-Schmidt orthogonalization process,

UNIT-IV

Orthogonal complements, Bessel's inequality, the adjoint of a linear operator, Least Squares Approximation, minimal solutions to systems of linear equations, Normal and self-adjoint operators, Orthogonal projections and Spectral theorem.

BOOKS RECOMMENDED:

1. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, *Linear Algebra* (4th Edition), Pearson, 2018.

BOOKS FOR REFERENCE:

1. Rao A R and Bhim Sankaram Linear Algebra Hindustan Publishing house.
2. Gilbert Strang, Linear Algebra and its Applications, Thomson, 2007.

CORE PAPER-XIII
COMPLEX ANALYSIS

Objectives: The objective of the courseis aimed to provide an introduction to the theories for functions of a complex variable. The concepts of analyticity and complex integration are presented. The Cauchy's theoremand its applications, the calculus of residues and its applications are discussed in detail.

Expected Outcomes: Students will be able to handle certain integrals not evaluated earlier and will know a technique for counting the zeros of polynomials. This course is prerequisite to many other advance analysis courses.

UNIT-I

Complex Numbers and Complex plane: Basic properties, convergence, Sets in the Complex plane, Functions on the Complex plane: Continuous functions, holomorphic functions, power series, Integration along curves.

UNIT-II

Cauchy's Theoremand Its Applications: Goursat's theorem, Local existence of primitives and Cauchy's theorem in a disc, Evaluation of some integrals, Cauchy's integral formulas.

UNIT-III

Morera’s theorem, Sequences of holomorphic functions, Holomorphic functions defined in terms of integrals, Schwarz reflection principle, Zeros and poles.

UNIT-IV

Meromorphic Functions and the Logarithm: The residue formula, Examples, Singularities and meromorphic functions, The argument principle and applications, The complex logarithm.

BOOKS RECOMMENDED:

1. Elias M. Stein & Rami Shakarchi, Complex Analysis, Princeton University press, Princeton and Oxford, 2003.

BOOKS FOR REFERENCE:

1. James Ward Brown and Ruel V. Churchill, *Complex Variables and Applications* (Eighth Edition), McGraw - Hill International Edition, 2009.
2. G. F. Simmons, *Introduction to Topology and Modern Analysis*, Mcgraw-Hill, Edition 2004.
3. Joseph Bak and Donald 1. Newman, *Complex analysis* (2ndEdition), Undergraduate Texts in Mathematics, Springer-Verlag New York, Inc., New York, 1997.

CORE PAPER-XIV
GROUP-THEORY-II

Objective: The objective of this course **i**s to be exposed to moreadvanced results in group theory after completing a basic course. The course introduces results on automorphism, commutator subgroup,group action Sylow theorems etc.

Expected Outcomes: The knowledge of automorphism helps to study more on field theory. Students learn on direct products, group actions , class equations and their applications with proof of all results . This course helps to opt for more advanced courses in algebra and linear classical groups.

UNIT-I

Automorphism, inner automorphism, automorphism groups, automorphism groups of finite and infinite cyclic groups,applications of factor groups to automorphism groups. Characteristic subgroups.

UNIT-II

Commutator subgroup and its properties, Properties of external direct products, the group of units modulo n as an external direct product, internal direct products, Fundamental Theorem of finite abelian groups.

UNIT-III

Group actions, stabilizers and kernels, permutation representation associated with a given group action, Application of group actions: Generalized Cayley's theorem, Index theorem.

UNIT-IV

Groups acting on themselves by conjugation, class equation and consequences, conjugacy in S_n, p - groups,Sylow's theorems and consequences, Cauchy's theorem, Simplicity of A_n for $n \geq 5$, non-simplicity tests.

BOOKS RECOMMENDED:

1. John B. Fraleigh, *A First Course in Abstract Algebra,* Narosa Publishing House, New Delhi.

2. Joseph A. Gallian *Contemporary Abstract Algebra* (4th Edition), Narosa Publishing House, New Delhi.

BOOK FOR REFERENCES:

1. M. Artin, *Abstract Algebra,* 2nd Ed., Pearson, 2011.
2. David S. Dummit and Richard M. Foote, *Abstract Algebra,* 3rd Ed., John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2004.
3. J.R. Durbin, *Modern Algebra,* John Wiley & Sons, New York Inc., 2000.

Discipline Specific Elective Paper-1

LINEAR PROGRAMMING

Objective: The objective of this course is to familiarize industrial problems to students with various methods of solving Linear Programming Problems, Transportation Problems, Assignment Problems and their applications. Also, students will know the application of linear Programming method in Game Theory.

Expected Outcomes: More knowledge on this topic in higher studies will help students to deal industrial models. This is also prerequisite for studying advanced courses in Nonlinear Programming Problems, Inventory Control Problem and Queuing Theory etc.

UNIT-I

Introduction to linear Programming problem, Theory of simplex method, optimality and unboundedness, the simplex alorithm, simplex method in tableau format, introduction to artificial variables, two-phase method, Big-M method and their comparison.

UNIT-II

Duality, formulation of the dual problem, primal-dual relationships, Fundamental Theorem of Duality, economic interpretation of the dual.

UNIT-III

Transportation problem and its mathematical formulation, northwest-corner method least cost method and Vogel approximation method for determination of starting basic solution, algorithm for solving transportation problem. Assignment problem and its mathematical formulation, Hungarian method for solving assignment problem.

UNIT-IV

Game theory: formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure, linear programming solution of games.

BOOKS RECOMMENDED:

1. Kanti Swarup, Operations Research, Sultan Chand & Sons, New Delhi. Books.

BOOKS FOR REFERENCE:

1. S. Hillier and G.J. Lieberman, *Introduction to Operations Research- Concepts and Cases* (9th Edition), TataMcGraw Hill, 2010.
2. Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali, *LinearProgramming and Network Flows* (2nd edition), John Wiley and Sons, India, 2004.
3. G. Hadley, *Linear Programming,* Narosa Publishing House, New Delhi, 2002.
4. Hamdy A. Taha, *Operations Research: An Introduction* (10th edition), Pearson, 2017.

Discipline Specific Elective Paper-II

Probability and Statistics

Objective: The objective of the course is to expertise the student to the extensive role of statistics in everyday life and computation, which has made this course a core course in all branches of mathematical and engineering sciences.

Expected Outcome: The students shall learn probability and statistics for various random variables, multivariate distributions, correlations and relations. He shall learn law of large numbers and shall be able to do basic numerical calculations.

UNIT-I

Probablity: Introduction, Sample spaces, Events, probability of events, rules of probability, conditional probability, independent events, Bayes's theorem,

Probability distributions and probability densities: random variables, probability distributions, continuous random variables, probability density functions, Multivariate distributions, joint distribution function, joint probability density function, marginal distributions, conditional distributions, conditional density, The theory in practice, data analysis, frequency distribution, class limits, class frequencies, class boundary, class interval, class mark, skewed data, multimodality, graphical representation of the data, measures of location and variability. Population, sample, parameters

UNIT-II

Mathematical Expectation:Introduction, expected value of random variable, moments, Chebyshev's theorem, moment generating functions, product moments, moments of linear combinations of random variables, conditional expectations, the theory in practice, measures of location, dispersion

UNIT-III

Special probability distributions: Discrete Uniform distribution, binomial distribution, Negative binomial, geometric, hypergeometric, poisson, multinomial distribution, multinomial. Special probability densities; Uniform distribution, gamma, exponential, gamma, chi-square, beta distribution, normal, normal approximation to binomial, bivariate normal, Functions of random variables, distribution function technique, transformation technique-one variable, several variables, moment generating function technique,

UNIT-IV

Sampling distributions: population distribution, random sample, sampling distribution of mean, Central Limit theorem, Sampling distribution of the mean: finite populations, chi-square, t, F distributions, regression and correlation: Bivariate regression, regression equation, Linear regression, method of least squares.

BOOKS RECOMMENDED:

1. Irwin Miller and Marylees Miller, *John E. Freund's Mathematical Statistics with Applications* (8^{th}Edition), Pearson, Asia, 2014.

BOOK FOR REFERENCES:

1. Robert V. Hogg, Joseph W. McKean and Allen T. Craig, *Introduction to Mathematical Statistics,* Pearson Education, Asia, 2007.

2.Alexander M. Mood, Franklin A. Graybill and Duane C. Boes, *Introduction to the Theory of Statistics,* (3rd Edition), Tata McGraw- Hill, Reprint 2007.

3. Sheldon Ross, *Introduction to Probability Models* (9th Edition), Academic Press, Indian Reprint, 2007.

Discipline Specific Elective Paper-III

DIFFERENTIAL GEOMETRY

Objective: After learning methods on curve tracing andAnalytic Geometry, the objective of this course is to teach Differential geometry of curves and surfaces which trains a student using tools in calculus to derive intrinsic properties of plain curves and space curves.

Expected Outcome: After completing this course a student will learn on serret-Frenet formulae, relation between tangent, normal and binormals, first and second fundamental forms and ideas on various curvatures. He has scope to take more advanced courses in surface theory and geometry.

UNIT-I

Theory of Space Curves: Space curves, Planer curves, Curvature, torsion and Serret-Frenet formulae. Osculating circles, Osculating circles and spheres. Existence of space curves.

UNIT-II

Evolutes and involutes of curves. Theory of Surfaces: Parametric curves on surfaces, surfaces of revolution, helicoids, Direction coefficients. First and second Fundamental forms.

UNIT-III

Principal and Gaussian curvatures. Lines of curvature, Euler's theorem. Rodrigue's formula,

Conjugate and Asymptotic lines. Developables: Developable associated with space curves and curves on surfaces, Minimal surfaces.

UNIT-IV

Geodesics: Canonical geodesic equations. Nature of geodesics on a surface of revolution. Clairaut's theorem. Normal property of geodesics. Torsion of a geodesic. Geodesic curvature. Gauss-Bonnet theorem. Surfaces of constant curvature.

BOOKS RECOMMENDED:

1. T.J. Willmore, *An Introduction to Differential Geometry,* Dover Publications, 2012.

BOOK FOR REFERENCES:

1. A. Pressley, Elementary Differential Geometry, Springer Internationl Edition, 2014.
2. O'Neill, *Elementary Differential Geometry,* 2nd Ed., Academic Press, 2006.
3. C.E. Weatherburn, *Differential Geometry of Three Dimensions,* Cambridge University Press 2003.
4. D.J. Struik, *Lectures on Classical Differential Geometry,* Dover Publications, 1988.

Discipline Specific Elective Paper-IV
NUMBER THEORY

Objective: The main objective of this course is to build up the basic theory of the integers, prime numbers and their primitive roots, the theory of congruence, quadratic reciprocity law and number theoretic functions, Fermat's last theorem, to acquire knowledge in cryptography specially in RSA encryption and decryption.

Expected Outcomes: Upon successful completion of this course students will able to know the basic definitions and theorems in number theory, to identify order of an integer, primitive roots, Euler's criterion,the Legendre symbol, Jacobi symboland theirproperties, to understand modular arithmetic number-theoretic functions and apply them to cryptography.

UNIT- I

Linear Diophantine equation,primecounting function, statement ofprime number theorem,Goldbach conjecture, linearcongruences, complete set of residues, Chinese remainder theorem,Fermat's littletheorem, Wilson's theorem.

UNIT-II

Number theoretic functions, sum and number of divisors, totally multiplicative functions, definition and properties ofthe Dirichlet product, the Mobius inversion formula, the greatest integer function, Euler's phi-function, Euler's theorem, reduced setofresidues, someproperties ofEuler's phi-function.

UNIT-III

Order of an integer modulo *n,* primitive roots for primes, composite numbers havingprimitive roots, Euler's criterion, the Legendre symbol, Jacobi symboland their properties, quadratic reciprocity, quadratic congruences with composite moduli.

UNIT-IV

Affine ciphers, Hill ciphers, publickey cryptography, RSA encryption anddecryption, theequation $x^2 + y^2 = z^2$,Fermat's Last Theorem.

BOOKS RECOMMENDED:

1. David M.Burton, *Elementary Number Theory* (6thEdition), TataMcGraw-Hill Edition, Indian reprint, 2007.

BOOK FOR REFERENCES:

1. Thomas Koshy, *ElementaryNumber Theory with Applications (*2nd Edition), Academic Press, 2007.
2. Neville Robinns, *Beginning Number Theory* (2ndEdition), Narosa Publishing House Pvt.Limited, Delhi,2007.

OR

Discipline Specific Elective Paper-IV

PROJECT

Guidelines for +3(CBCS) Under Graduate(B.A./B.Sc.) Mathematics(Honours) Project

1. Any student registering for doing project is required to inform the HOD, Mathematics the name of his/her project supervisor(s) at the time of pre-registration.
2. By the last date of add and drop, the student must submit the "Project Registration Form'', appended as Annexure-I to this document, to the HOD, Mathematics. This form requires a project title, the signature of the student, signature(s) of the supervisor(s) and the signature of the HOD, Mathematics of the college/university.
3. The project supervisor(s) should normally be a faculty member(s) of the Department of Mathematics and the topic of the project should be relevant to Mathematical Sciences. If a student desires to have a Project Supervisor from another department of the institute, the prior approval for the same should be sought from the HOD, Mathematics.
4. A student may have at the most two Project Supervisors. If a student desires to have two supervisors, at least one of these should be from the Department of Mathematics.
5. The student(s) will be required to submit one progress report and a final report of the Project to the HOD, Mathematics. The progress report is to be submitted in the sixth week of the semester in which the project is undertaken. The hard copy and an electronic version of the final report of the project should be submitted two weeks before the end semester examination of the sixth semester. In addition the student will be required to make an oral presentation in front of a committee **(**Under Graduate (B.A./B.Sc.) Mathematics (Honours) Project committee of the college in which supervisor is one of the members) constituted for this purpose by the Department of Mathematics of the college.
6. The student is expected to devote about 100 hours. The project will be evaluated by a committee of faculty members at the end of the sixth semester. The committee will be constituted by the Under Graduate (B.A./B.Sc.) Mathematics(Honours) Project committee of the college keeping in mind the areas of project they will cover.

7. In each semester the grade of a student will be awarded by the committee in consultation with his/her project supervisor(s). The project is evaluated on the basis of the following components: First Progress Reports: 20%; second/Final Report: 30%; Presentation: 30%; Viva:20%.
8. Project progress reports should normally be no longer than 250 words and final report should not be longer than 40 A4 size pages in double spacing. Each final project report need to contain the following: (i) Abstract (ii) Table of contents (iii)Review of literature (iv) Main text(v) List of references. It may be desirable to arrange the main text as an introduction, the main body and conclusions.

GUIDELINES FOR STRUCTURING CONTENTS

Sequence of Contents:

The following sequence for the thesis organization should be followed:

(i) Preliminaries	Title Page
	Certificate
	Abstract/Synopsis
	Acknowledgement and/ or Dedication
	Table of Contents
	List of Figures, Tables, Illustrations,
	Symbols, etc (wherever applicable)
(ii) Text of Thesis	Introduction
	The body of the thesis, summary and conclusions
(iii) Reference Material	List of References, Bibliography
(iv) Appendices	

NOTE:

1. *Synopsis/Abstract* should be self-complete and contain no citations for which the thesis has to be referred.

2. The Text of the Thesis

(a) Introduction:

Introduction may be the first chapter or its first major division. In either case, it should contain a brief statement of the problem investigated. It should outline the scope, aim, general character of the research and the reasons for the student's interest in the problem.

(b) The body of Thesis

This is the substance of the dissertation inclusive of all divisions, subdivisions, tables, figures, etc.

(c) Summary and conclusions

If required, these are given as the last major division (chapter) of the text. A further and final subdivision titled "*Scope* for Further Work" may follow.

(d) Reference material

The list of references should appear as a consolidated list with references listed either alphabetically or sequentially as they appear in the text of the thesis.

For referencing an article in a scientific journal the suggested format should contain the following information: authors, title, name of journal, volume number, page numbers and year. For referencing an article published in a book, the suggested format should contain, authors, the title of the book, editors, publisher, year, page number of the article in the book being referred to. For referencing a thesis the suggested format should contain, author, the title of thesis, where thesis was submitted or awarded, year.

ANNEXURE-I

Department of Mathematics

Project Registration Form

Name of the college/university:

Name of the student:

Roll No. :

e-mail :

Name of the supervisor(s):

Department(s):

e-mail(s):

Title of the Project:

Signature of the Student:

Signature of supervisor(s): (i)

(ii)

Signature of HOD, Mathematics:

GENERIC ELECTIVES (TWO PAPER CHOICE)

Generic Elective Paper I

CALCULUS AND DIFFERENTIAL EQUATIONS

Objective: Calculus invented by Newton and Leibnitz is a powerful analytical tool to solve mathematical problems which arise in all branches of science and engineering. The main emphasis of this course is to equip the student with necessary analytic and technical skills to handle problems of a mathematical nature as well as practical problems using calculus and differential equation. The aim should be to expose the students to basic ideas quickly without much theoretical emphasis with importance on applications.

Excepted Outcomes: After completing the course, students are expected to be able to apply knowledge of calculus and differential equations in the areas of their own interest.

UNIT-I

Curvature, Asymptotes, Tracing of Curves (Catenary, Cycloid, Folium of Descartes), Rectification, Quadrature, Elementary ideas about Sphere, Cones, Cylinders and Conicoids.

UNIT-II

Reviewoflimits,continuityanddifferentiabilityoffunctionsofonevariable and their properties, Rolle's theorem, Mean value theorems, Taylor's theorem with Lagrange's theorem and Cauchy's form of remainder, Taylor's series, Maclaurin's series of $\sin x, \cos x, e^{s}, \log(1+x)$, $(1+x)^{N}$, L' Hospital's Rule, other Intermediate forms.

UNIT-III

Limitand Continuity of functions of several variables, Partial derivatives, Partial derivatives of higher orders, Homogeneous functions, Change of variables, Mean value theorem,Taylors theorem and Maclaurin's theorem for functions of two variables (statements &applications), Maxima and Minima of functions of two and three variables, Implicit functions, Lagrange's multipliers (Formulae & its applications), Concepts of Multiple integrals & its applications.

UNIT-IV

Ordinary Differential Equations of order one and degree one (variables separable, homogeneous, exact and linear). Equations of order one but higher degree. Second order linear equations with constant coefficients, homogeneous forms, Second order equations with variable coefficients, Variation of parameters.

BOOKS RECOMMENDED:

1. Shanti Narayan, P. K. Mittal, Differential Calculus, S. Chand, 2014.
2. Shanti Narayan, P. K. Mittal, Integral Calculus, S. Chand, 2014.
3. S.C. Mallik and S. Arora-Mathematical Analysis, New Age InternationalPublications.
4. J. Sinharoy and S. Padhy: A Course of Ordinary and Partial Differential Equations, Kalyani Publishers.

BOOK FOR REFERENCES:

1. H.Anton,I.Bivensand S.Davis,*Calculus*,10thEd.,JohnWileyand Sons(Asia)P.Ltd.,Singapore, 2002.
2. Shanti Narayan and P.K. Mittal-Analytical Solid Geometry, S. Chand & Company Pvt. Ltd., New Delhi.
3. Martin Braun-Differential Equations and their Applications-Martin Braun, Springer International.
4. B. P.AcharyaandD. C.Sahu:AnalyticalGeometryofQuadraticSurfaces,KalyaniPublishers.

Generic Elective Paper II

ALGEBRA

Objective: This is a preliminary course for the basic courses in mathematics like, abstract algebra and linear algebra. The objective is to acquaint students with the properties of natural numbers i.e. Euclidean algorithm, congruence relation, fundamental theorem of arithmetic, etc. The basics of linear algebra i.e. vector spaces, matrices are introduced here.

Expected Outcomes: Theacquired knowledge will help students to study further courses in mathematics like, group theory, ring theory and field theory and linear algebra. It has applications not only in higher mathematics but also in other science subjects like computer science, statistics, physics, chemistry etc.

UNIT-I

Sets, relations,Equivalence relations, partial ordering, well ordering, Functions, Composition of functions, Invertible functions, One to one correspondence and cardinality of a set, statements, compound statements, proofs in Mathematics,Truth tables, Algebra of propositions, logical arguments

UNIT-I

Well-ordering property of positive integers, Division algorithm, Divisibility and Euclidean algorithm, Congruence relation between integers, Principles of Mathematical Induction, statement of Fundamental Theorem of Arithmetic.

UNIT-III

Matrices, algebra of matrices, determinants, fundamental properties, minors and cofactors, product of determinant, adjoint and inverse of a matrix, Rank and nullity of a matrix, Systems of linear equations, row reduction and echelon forms, solution sets of linear systems, applications of linear systems,.

UNIT-IV

Vector spaces and subspaces, examples, linear independence, linear dependence, basis, dimension, examples, Introduction to linear transformations, matrix representation of a linear transformation, Eigen values, Eigen vectors of amatrix.

BOOKS RECOMMENDED:

1. Edgar G. Goodaire and Michael M. Parmenter, Discrete Mathematics with Graph Theory,

3rd Ed., Pearson Education (Singapore) P. Ltd., Indian Reprint, 2005.

2. V Krishna Murthy, V P Mainra, J L Arora, An Introduction to Linear Algebra , Affiliated East-West Press Pvt. Ltd

BOOKS FOR REFERENCE:

1. DavidC.Lay,LinearAlgebraanditsApplications,3rdEd.,PearsonEducationAsia,Indian Reprint,2007.

2. B S Vatsa and Suchi Vatsa Theory of Matrices New age International third edition 2010.

3. Ward Cheney, David kincaid. Linear algebra theory and applications, Jones and Bartlett ,2010.

OR

GENERIC ELECTIVES (FOR FOUR PAPERS CHOICE)

Generic Elective Paper III

REAL ANALYSIS

Objective: The objective of the course is to have the knowledge on basic properties of the field of real numbers, studying Bolzano-Weiersstrass Theorem, sequences and convergence of sequences, series of real numbers and its convergence etc. This is one of the core courses essential to start doing mathematics.

Expected Outcome: On successful completion of this course, students will be able to handle fundamental properties of the real numbers that lead to the formal development of real analysis and understand limits and their use in sequences, series, differentiation and integration. Students will appreciate how abstract ideas and rigorous methods in mathematical analysis can be applied to important practical problems.

UNIT-I

Review of Algebraic and Order Properties of *R,* s-neighborhood of a point in *R,* Idea of countable sets, uncountable sets and uncountability of *R,* Bounded above sets, Bounded below sets, Bounded Sets, Unbounded sets, Suprema and Infima, The Completeness Property of *R,* The Archimedean Property, Density of Rational (and Irrational) numbers in *R.*

UNIT-II

Intervals, Interior point, Open Sets, Closed sets, Limit points of a set , Illustrations of Bolzano-Weierstrass theorem for sets, closure, interior and boundary of a set. Sequences, Bounded sequence, Convergent sequence, Limit of a sequence. Limit Theorems ,Monotone Sequences, Monotone Convergence Theorem. Subsequences, Divergence Criteria, Monotone Subsequence Theorem (statement only). Bolzano Weierstrass Theorem for Sequences, Cauchy sequence, Cauchy's Convergence Criterion.

UNIT-III

Infinite series, convergence and divergence of infinite series, Cauchy Criterion,Tests for convergence: Comparison test, Limit Comparison test, Ratio Test, Cauchy's nth root test, Integral test, Alternating series, Leibniz test, Absolute and Conditional convergence.

UNIT-IV

Sequence and Series of functions, point-wise and uniform convergences, M_n test, M test, statement of results about uniform convergence, differentiability and integrability of function, power series and radius of convergence.

BOOKS RECOMMENDED:

1. S.C. Mallik and S. Arora-Mathematical Analysis, New Age InternationalPublications.
2. G. Das and S. Pattanayak, Fundamentals of Mathematical Analysis, TMH Publishing Co.

BOOKS FOR REFERENCE:

1.R.G. Bartle and D. R. Sherbert, Introduction to Real Analysis(3rd Edition), John Wiley and Sons (Asia) Pvt. Ltd., Singapore,2002.

2. A.Kumar, S. Kumaresan, *A basic course in Real Analysis*, CRC Press, 2014.
3. BrianS.*Thomson*,Andrew.M.*Bruckner*,andJudithB.*Bruckner*,*ElementaryReal Analysis,* Prentice Hall,2001.
4. Gerald G. Bilodeau, Paul R. Thie, G.E. Keough, *An Introductionto Analysis,* Jones & Bartlett, Second Edition, 2010.

Generic Elective Paper IV

NUMERICAL METHODS

Objective: Calculation of error and approximation is a necessity in all real life, industrial and scientific computing. The objective of this course is to acquaint students with various numerical methods of finding solution of different type of problems, which arises in different branches of science such as locating roots of equations, finding solution of nonlinear equations, systems of linear equations, differential equations, Interpolation, differentiation, evaluating integration.

Expected Outcome: Students can handle physical problems to find an approximated solution. After getting trained a student can opt for advance courses in Numerical analysis in higher mathematics. Use of good mathematical software will help in getting the accuracy one need from the computer and can assess the reliability of the numerical results, and determine the effect of round off error or loss of significance.

UNIT-I

Algorithms, Convergence, Bisection method, False position method, Fixed pointiterationmethod, Newton’s method, Secant method.

Gauss Elimination and Gauss Jordan methods, LU decomposition, Gauss-Jacobi, Gauss-Siedel.

UNIT-II

Lagrange and Newton interpolation: linear and higher order, finite differenceoperators.

UNIT-III

Numerical differentiation: forward difference, backward difference and centralDifference.

UNIT-IV

Integration: trapezoidal rule, Simpson’s rule, Euler’smethod, Runge-Kutta methods of orders two and four.

BOOKS RECOMMENDED:

1. M.K. Jain, S.R.K. Iyengar and R.K. Jain, *Numerical Methods for Scientific andEngineering Computation*, 5th Ed., New age International Publisher, India,2007.

BOOKS FOR REFERENCE:

1. S. S. Sastry, Introductory method for Numerical Analysis, PHI New Delhi,2012.
2. S. D. Conte and Carl De Boor, Elementary Numerical Analysis, Mc Graw Hill, 1980.

STRUCTURE OF THE +3 UNDER GRADUATE (B.A / B.Sc)
MATHEMATICS (PASS) SYLLABUS
BASED ON CHOICE BASED CREDIT SYSTEM (CBCS)

Semester	Course Number	Title of the Course	Number of credits assigned to the course		Total Credits
			Theory	Practical(P)/ Tutorial((T)	
		DSC 4 PAPERS			
	MATH- DSC-1	Calculus and Differential equations	5	1	6
	MATH-DSC2	Algebra	5	1	6
	MATH-DSC-3	Real Analysis	5	1	6
	MATH-DSC-4	Numerical Methods	5	1	6

DSE 2 PAPERS					
	MATH- DSE-1	Group Theory	5	1	6
	MATH-DSE-2	Linear Programming	5	1	6
				TOTAL	36

B.A./B.SC.(PASS)-MATHEMATICS

MATHEMATICS PAPERS FOR PASS STUDENTS

Discipline Specific Core – 4 papers

Discipline Specific Elective – 2 papers

Marks per paper – Mid term : 20 marks, End term : 80 marks

Total – 100 marks Credit per paper – 6

Teaching hours per paper – 50 hours Theory classes + 10 hours tutorial

Discipline Specific Core Paper I

CALCULUS AND DIFFERENTIAL EQUATIONS

Objective: Calculus invented by Newton and Leibnitz is powerful analytical tool to solve mathematical problems which arise in all branches of science and engineering. The main emphasis of this course is to equip the student with necessary analytic and technical skills to handle problems of a mathematical nature as well as practical problems using calculus and differential equation. The aim should be to expose the students to basic ideas quickly without much theoretical emphasis with importance on applications.

Excepted Outcomes: After completing the course, students are expected to be able to apply knowledge of calculus and differential equations in the areas of their own interest.

UNIT-I

Curvature, Asymptotes, Tracing of Curves (Catenary, Cycloid, Folium of Descartes), Rectification, Quadrature, Elementary ideas about Sphere, Cones, Cylinders and Conicoids.

UNIT-II

Reviewoflimits,continuityanddifferentiabilityoffunctionsofonevariable andtheirproperties, Rolle's theorem, Mean value theorems, Taylor's theorem with Lagrange's theorem and Cauchy's form of remainder, Taylor's series, Maclaurin's series of $\sin x, \cos x, e^{s}, \log(1+x), (1+x)^{N}$, L' Hospital's Rule, other Intermediate forms.

UNIT-III

Limitand Continuity of functions of several variables, Partial derivatives, Partial derivatives of higher orders, Homogeneous functions,Change of variables,Mean value theorem,TaylorstheoremandMaclaurinstheorem for functions of two variables(statements &applications), Maxima and Minima of functions of two and three variables, Implicit functions, Lagranges multipliers (Formulae & its applications), Concepts of Multiple integrals & its applications.

UNIT-IV

OrdinaryDifferentialEquationsoforderoneanddegreeone(variablesseparable,homogeneous,exactand linear).Equationsoforderonebuthigherdegree.Secondorderlinearequationswithconstantcoefficients, homogeneousforms,Secondorderequationswithvariablecoefficients,Variationofparameters.

BOOKS RECOMMENDED:

1. Shanti Narayan, P. K. Mittal, Differential Calculus, S. Chand, 2014.

2. Shanti Narayan, P. K. Mittal, Integral Calculus, S. Chand, 2014.

3. S.C. Mallik and S. Arora-Mathematical Analysis, New Age InternationalPublications.

4. J. Sinharoy and S. Padhy: A Course of Ordinary and Partial Differential Equations, Kalyani Publishers.

BOOKS FOR REFERENCE:

1. H.Anton,I.Bivensand S.Davis,*Calculus*,10thEd.,JohnWileyand Sons(Asia)P.Ltd.,Singapore, 2002.

2. Shanti Narayan and P.K. Mittal-Analytical Solid Geometry, S. Chand & Company Pvt. Ltd., New Delhi.

3. Martin Braun-Differential Equations and their Applications-Martin Braun, Springer International.

4. B. P.AcharyaandD. C.Sahu:AnalyticalGeometryofQuadraticSurfaces,KalyaniPublisher

Discipline Specific Core Paper II

ALGEBRA

Objective: This is a preliminary course for the basic courses in mathematics like, abstract algebra and linear algebra. The objective is to acquaint students with the properties of natural numbers i.e. Euclidean algorithm, congruence relation, fundamental theorem of arithmetic, etc. The basics of linear algebra i.e. vector spaces, matrices are introduced here.

Expected Outcomes: Theacquired knowledge will help students to study further courses in mathematics like, group theory, ring theory and field theory and linear algebra. It has applications not only in higher mathematics but also in other science subjects like computer science, statistics, physics, chemistry etc.

UNIT-I

Sets,relations,Equivalence relations,partial ordering,well ordering, Functions, Composition of functions, Invertible functions, One to one correspondence and cardinality of a set, statements, compound statements,proofs in Mathematics,Truth tables, Algebra of propositions,logical arguments

UNIT-II

Well-ordering property of positive integers, Division algorithm, Divisibility and Euclidean algorithm, Congruence relation between integers, Principles of Mathematical Induction, statement of Fundamental Theorem of Arithmetic.

UNIT-III

Matrices, algebra of matrices , determinants, fundamental properties, minors and cofactors, product of determinant, adjoint and inverse of a matrix, Rank and nullity of a matrix, Systems of linear equations, row reduction and echelon forms, solution sets of linear systems, applications of linear systems,.

UNIT-IV

Vector spaces and subspaces, examples, linear independence, linear dependence, basis, dimension, examples, Introduction to linear transformations, ,matrix representation of a linear transformation,Eigen values, Eigen vectors of amatrix.

BOOKS RECOMMENDED:

1. Edgar G. Goodaire and Michael M. Parmenter, Discrete Mathematics with Graph Theory, 3rd Ed., Pearson Education (Singapore) P. Ltd., Indian Reprint, 2005.
2. V Krishna Murthy, V P Mainra, J L Arora, An Introduction to Linear Algebra , Affiliated East-West Press Pvt. Ltd

BOOKS FOR REFERENCE:

1. DavidC.Lay,LinearAlgebraanditsApplications,3rdEd.,PearsonEducationAsia,Indian Reprint, 2007.

2. B S Vatsa and Suchi Vatsa Theory of Matrices New age International third edition, 2010.

3. Ward Cheney, David Kincaid. Linear algebra theory and applications ,Jones and Bartlett, 2010.

Discipline Specific Core Paper III

REAL ANALYSIS

Objective: The objective of the course isto have the knowledge on basic properties of the field of real numbers, studying Bolzano-Weiersstrass Theorem , sequences and convergence of sequences, series of real numbers and its convergence etc. This is one of the core courses essential to start doing mathematics.

Expected Outcome: On successful completion of this course, students will be able to handle fundamental properties of the real numbers that lead to the formal development of real analysis and understand limits and their use in sequences, series, differentiation and integration. Students will appreciate how abstract ideas and rigorous methods in mathematical analysis can be applied to important practical problems.

UNIT-I

Review of Algebraic and Order Properties of *R,* s-neighborhood of a point in *R,* Idea of countable sets, uncountable sets and uncountability of *R,* Bounded above sets, Bounded below sets, Bounded Sets, Unbounded sets, Suprema and Infima, The Completeness Property of *R,* The Archimedean Property, Density of Rational (and Irrational) numbers in *R.*

UNIT-II

Intervals, Interior point, Open Sets, Closed sets, Limit points of a set , Illustrations of Bolzano-Weierstrass theorem for sets, closure, interior and boundary of a set. Sequences, Bounded sequence, Convergent sequence, Limit of a sequence. Limit Theorems ,Monotone Sequences, Monotone Convergence Theorem. Subsequences, Divergence Criteria, Monotone Subsequence Theorem (statement only). Bolzano Weierstrass Theorem for Sequences, Cauchy sequence, Cauchy's Convergence Criterion.

UNIT-III

Infinite series, convergence and divergence of infinite series, Cauchy Criterion, Tests for convergence: Comparison test, Limit Comparison test, Ratio Test, Cauchy's nth root test, Integral test, Alternating series, Leibniz test, Absolute and Conditional convergence.

UNIT-IV

Sequence and Series of functions, pointwise and uniform convergences, M_n test, M test, statement of results about uniform convergence, differentiability and integrability of function, power series and radius of convergence.

BOOKS RECOMMENDED:

1. S.C. Mallik and S. Arora-Mathematical Analysis, New Age InternationalPublications.
2. G. Das and S. Pattanayak, Fundamentals of Mathematical Analysis, TMH Publishing Co.

BOOKS FOR REFERENCE:

1. R.G. Bartle and D. R. Sherbert, Introduction to Real Analysis(3rd Edition), John Wiley and Sons (Asia) Pvt. Ltd., Singapore,2002.
2. A.Kumar, S. Kumaresan, *A basic course in Real Analysis*, CRC Press, 2014.
3. BrianS.*Thomson*,Andrew.M.*Bruckner*,andJudithB.*Bruckner*,*ElementaryReal Analysis,* Prentice Hall,2001.
4. Gerald G. Bilodeau , Paul R. Thie, G.E. Keough, *An Introductionto Analysis,* Jones & Bartlett, Second Edition, 2010.

Discipline Specific Core Paper IV

NUMERICAL METHODS

Objective: Calculation of error and approximation is a necessity in all real life, industrial and scientific computing. The objective of this course is to acquaint students with various numerical methods of finding solution of different type of problems, which arises in different branches of science such as locating roots of equations, finding solution of nonlinear equations, systems of linear equations, differential equations, Interpolation, differentiation, evaluating integration.

Expected Outcome: Students can handle physical problems to find an approximated solution. After getting trained a student can opt for advance courses in Numerical analysis in higher mathematics. Use of good mathematical software will help in getting the accuracy one need from the computer and can assess the reliability of the numerical results, and determine the effect of round off error or loss of significance.

UNIT-I

Algorithms, Convergence, Bisection method, False position method, Fixed pointiterationmethod, Newton's method, Secant method.

Gauss Elimination and Gauss Jordan methods, LU decomposition, Gauss-Jacobi, Gauss-Siedel.

UNIT-II

Lagrange and Newton interpolation: linear and higher order, finite differenceoperators.

UNIT-III

Numerical differentiation: forward difference, backward difference and centralDifference.

UNIT-IV

Integration: trapezoidal rule, Simpson's rule, Euler'smethod, Runge-Kutta methods of orders two and four.

BOOKS RECOMMENDED:

1. M.K. Jain, S.R.K. Iyengar and R.K. Jain, *Numerical Methods for Scientific andEngineering Computation*, 5th Ed., New age International Publisher, India,2007.

BOOKS FOR REFERENCE:

1. S. S. Sastry, Introductory method for Numerical Analysis, PHI New Delhi,2012.
2. S. D. Conte and Carl De Boor, Elementary Numerical Analysis, Mc Graw Hill, 1980.

Discipline Specific Elective Paper –I

GROUP THEORY

Objective: Group theory is one of the building blocks of modern algebra. Objective of this course is to introduce students to basic concepts of group theory and examples of groups and their properties. This course will lead to future basic courses in advanced mathematics, such as Group theory-II and ring theory.

Expected Outcomes: A student learning this course gets idea on concept and examples of groups and their properties . He understands cyclic groups, permutation groups, normal subgroups and related results. After this course he can opt for courses in ring theory, field theory, commutative algebras, linear classical groups etc. and can be apply this knowledge to problems in physics, computer science, economics and engineering.

UNIT-I

Symmetries of a square, Dihedral groups, definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), elementary properties of groups, Subgroups and examples of subgroups, centralizer, normalizer, center of a group,

UNIT-II

Product of two subgroups, Properties of cyclic groups, classification of subgroups of cyclic groups,Cycle notation for permutations, properties of permutations, even and odd permutations, alternating group,

UNIT-III

Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem, external direct product of a finite number of groups, normal subgroups, factor groups.

UNIT-IV

Cauchy's theorem for finite abelian groups, group homomorphisms, properties of homomorphisms, Cayley's theorem, properties of isomorphisms, first, second and third isomorphism theorems.

BOOKS RECOMMENDED:

1. Joseph A. Gallian, *Contemporary Abstract Algebra* (4th Edition), Narosa Publishing House, New Delhi,
2. John B. Fraleigh, *A First Course in Abstract Algebra,* 7th Ed., Pearson, 2002.

BOOK FOR REFERENCES:

1. M. Artin*, Abstract Algebra,* 2nd Ed., Pearson, 2011.
2. Joseph 1. Rotman, *An Introduction to the Theory of Groups,* 4th Ed., Springer Verlag, 1995.
3. I. N. Herstein, *Topics in Algebra,* Wiley Eastern Limited, India, 1975.

Discipline Specific Elective Paper –II

LINEAR PROGRAMMING

Objective: The objective of this course is to familiarize industrial problems to students with various methods of solving Linear Programming Problems, Transportation Problems, Assignment Problems and their applications. Also, students will know the application of linear Programming method in Game Theory.

Expected Outcomes: More knowledge on this topic in higher studies will help students to deal industrial models. This is also prerequisite for studying advanced courses in Nonlinear Programming Problems, Inventory Control Problem and Queuing Theory etc.

UNIT-I

Introduction to linear Programming problem, Theory of simplex method, optimality and unboundedness, the simplex alorithm, simplex method in tableau format, introduction to artificial variables, two-phase method, Big-M method and their comparison.

UNIT-II

Duality, formulation of the dual problem, primal-dual relationships, Fundamental Theorem of Duality, economic interpretation of the dual.

UNIT-III

Transportation problem and its mathematical formulation, northwest-corner method least cost method and Vogel approximation method for determination of starting basic solution, algorithm for solving transportation problem. Assignment problem and its mathematical formulation, Hungarian method for solving assignment problem.

UNIT-IV

Game theory: formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure, linear programming solution of games.

BOOKS RECOMMENDED:

1. Kanti Swarup, Operations Research, Sultan Chand & Sons, New Delhi. Books.

BOOKS FOR REFERENCE:

1. Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali, *LinearProgramming and Network Flows* (2nd edition), John Wiley and Sons, India, 2004.
2. Hillier and G.J. Lieberman, *Introduction to Operations Research- Concepts and Cases* (9th

Edition), TataMcGraw Hill, 2010.

3. G. Hadley, *Linear Programming,* Narosa Publishing House, New Delhi, 2002.
4. Hamdy A. Taha, *Operations Research: An Introduction* (10th edition), Pearson, 2017

SKILL ENHANCEMENT COMPULSORY COURSES (SECC)

Optional for SECC II paper

Skill Enhancement Compulsory Courses (Option1)

COMPUTER GRAPHICS

Development of computer Graphics: Raster Scan and Random Scan graphics storages, displays processors and character generators, colour display techniques, interactive input/output devices. Points, lines and curves: Scan conversion, line-drawing algorithms, circle and ellipse generation, conic-section generation, polygon filling anti aliasing. Two-dimensional viewing: Coordinate systems, linear transformations, line and polygon clipping algorithms.

<u>Books Recommended:</u>

1. D. Hearn and M.P. Baker-Computer Graphics, 2nd Ed., PrenticeHall of India, 2004.
2. J. D. Foley, A van Dam, S.K. Feiner and J.F. Hughes-Computer Graphics: Principals and Practices, 2nd Ed., Addison-Wesley, MA, 1990.
3. D. F. Rogers-Procedural Elements in Computer Graphics, 2nd Ed., McGraw Hill Book Company, 2001.

4. D. F. Rogers and A. J. Admas-Mathematical Elements in Computer Graphics, 2nd Ed., McGraw Hill Book Company, 1990.

SKILL ENHANCEMENT COURSES (Option2)-

INFORMATION SECURITY

Overview of Security: Protection versus security; aspects of security data integrity, data availability, privacy; security problems, user authentication, Orange Book. Security Threats: Program threats, worms, viruses, Trojan horse, trap door, stack and buffer over flow; system threats- intruders; communication threats- tapping and piracy. Security Mechanisms: Intrusion detection, auditing and logging, tripwire, system-call monitoring.

Books Recommended:

1. C. Pfleeger and S. L. Pfleeger-Security in Computing, 3rd Ed., Prentice-Hall of India, 2007.
2. D. Gollmann-Computer Security, John Wiley and Sons, NY, 2002.
3. J. Piwprzyk, T. Hardjono and J. Seberry-Fundamentals of Computer Security, Springer-Verlag Berlin, 2003. 335
4. J.M. Kizza-Computer Network Security, Springer, 2007.

5. M. Merkow and J. Breithaupt-Information Security: Principles and Practices, Pearson Education, 2006.

Training Programmes to be Imparted

1. There should be training programs in MATLAB/ PYTHON/R/ MATHEMATICA software for all college teachers to acquaint the teachers on state of the art. Experts from Indian Statistical Institute Kolkata and nearby IIT's should be invited for the programs to ensure quality.

2. The faculty members in colleges/universities should be trained in the following courses at University or any Institute of Higher Learning.

a) Advanced Group Theory
b) Advanced Ring Theory
c) Differential Equations & Mathematical Modeling
d) Mathematical Finance
e) Object Oriented Programming in C++

f) Computer Graphics
g) Information Security

3. Emphasis may be given for implementation of the programs as listed in the courses with Practical.

4. College/ Universities should be provided with the recommended set of books in adequate numbers.

5. There should be frequent visits to colleges/ Universities offering crash courses to initiate some of the new courses.

Required Equipment/Technical Experts

The following equipment /software are to be provided to colleges / universities for smooth running of practical/ project:

1. There should be funding to Computer Lab with minimum of 15 computer systems for 30 students with licensed MATLAB/ PYTHON/R/MATHEMATICA software.
2. At least one computer programmer must be assigned in computer labs during practical sessions.